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Development of calibration and digital filtering procedures for a 5-component strain-gage balance

Young, Shih-Yih, M.S.

The University of Texas at Arlington, 1991
DEVELOPMENT OF CALIBRATION AND DIGITAL FILTERING PROCEDURES
FOR A 5-COMPONENT STRAIN-GAGE BALANCE

The members of the Committee approve the masters
thesis of Shih-Yih Young

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Frank K. Lu
DEVELOPMENT OF CALIBRATION AND DIGITAL FILTERING PROCEDURES
FOR A 5-COMPONENT STRAIN-GAGE BALANCE

by

SHIH-YIH YOUNG

Presented to the Faculty of the Graduate School of
The University of Texas at Arlington in Partial Fulfillment
of the Requirements
for the degree of

MASTER OF SCIENCE IN AEROSPACE ENGINEERING

THE UNIVERSITY OF TEXAS AT ARLINGTON

August 1991
ACKNOWLEDGEMENTS

I would like to express my sincere appreciation and thanks to my advisor, Dr. Wilson, for giving me the opportunity to work on this project and providing much of his time and effort during the course of this project.

I want to express my appreciation to my thesis committee composed of Dr. Wilson, Dr. Seath and Dr. Lu. I would also like to thank Mr. Cooksey and Mr. Stuessy for helping me during the course of this work.

July 19, 1991
ABSTRACT

DEVELOPMENT OF CALIBRATION AND DIGITAL FILTERING PROCEDURES FOR A 5-COMPONENT STRAIN-GAGE BALANCE

Publication No.______________

Shih-Yih Young, M.S.

The University of Texas at Arlington, 1991

Supervising Professor: Donald R. Wilson

A 5-component strain-gage balance was developed and calibrated for acquiring transonic wind tunnel force and moment data. A Fast Fourier Transform code was developed to filter the oscillations in the raw balance data as part of the data reduction procedure. This thesis presents the calibration procedure, the filter theory, and wind tunnel data for two representative helicopter rotor blade sections at low transonic Mach numbers.
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<th><strong>Definition</strong></th>
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<tbody>
<tr>
<td>A</td>
<td>Interaction force and moment matrix</td>
</tr>
<tr>
<td>b</td>
<td>Span</td>
</tr>
<tr>
<td>c</td>
<td>Chord</td>
</tr>
<tr>
<td>$C_{D0}$</td>
<td>Minimum drag coefficient</td>
</tr>
<tr>
<td>$C_D$</td>
<td>Drag coefficient</td>
</tr>
<tr>
<td>$C_L$</td>
<td>Lift coefficient</td>
</tr>
<tr>
<td>$C_{L_{max}}$</td>
<td>Maximum lift coefficient</td>
</tr>
<tr>
<td>$C_L\alpha$</td>
<td>Variation of lift coefficient with angle of attack</td>
</tr>
<tr>
<td>$C_I$</td>
<td>Rolling moment coefficient</td>
</tr>
<tr>
<td>$C_M$</td>
<td>Pitching moment coefficient</td>
</tr>
<tr>
<td>$C_{m1}$</td>
<td>Pitching moment coefficient with respect to the center of the chord</td>
</tr>
<tr>
<td>$C_{m2}$</td>
<td>Pitching moment coefficient with respect to the leading edge</td>
</tr>
<tr>
<td>$C_n$</td>
<td>Yawing moment coefficient</td>
</tr>
<tr>
<td>CF</td>
<td>Chord force</td>
</tr>
<tr>
<td>D</td>
<td>Drag</td>
</tr>
<tr>
<td>L</td>
<td>Lift</td>
</tr>
<tr>
<td>l</td>
<td>Rolling moment</td>
</tr>
<tr>
<td>M</td>
<td>Mach number</td>
</tr>
</tbody>
</table>
MX  Root bending moment
MZ  Uncorrected yawing moment
m   Pitching moment
NF  Normal force
n   Yawing moment
R1  Resistance for channel 1
R2  Resistance for channel 2
R3  Resistance for channel 3
R4  Resistance for channel 4
R5  Resistance for channel 5
p   Static pressure
Re  Reynolds number
S   Model surface area
t   Time
ρ   Density
α  Angle of attack
γ  Specific heat ratio
CHAPTER I

INTRODUCTION

Early balance equipment was located external to the wind tunnel and the model loads were carried out of the tunnel to the balance by struts and wires attached to the wings and fuselage. The external balance is still used today in low speed wind tunnels, but it has many disadvantages. A summary of these disadvantages follows.

For a model with struts attached to the wings and fuselage, the drag measurement by an external balance is a sum of the following contributions: drag of the model, drag due to strut flow interference on the wings, drag due to strut interference on fuselage flow, drag due to model interference on strut flow, and free air drag of the strut. The only drag measurement desired out of this group is the drag of the model. The other drag contributions may be evaluated but only by a tedious process of elimination described by Pope and Harper (Ref. 1). Other measurements such as lift and moments are also affected by the presence of the strut supports. Unfortunately, these interference effects are seldom fully determined due to time and extra data analysis required.
In the 1950's, internal model balances using sting supports were developed (Ref 1). Supersonic tunnel testing required development of this internal balance in order to free the model from strut-induced shock wave interference. Once the internal balance was developed, it partially replaced the external balance in wind tunnel testing.

When a load is applied to the internal balance, strain-gage flexures distort. The gages attached to these flexures change resistance under distortion creating an electrical signal proportional to the load applied. The advantage of the internal balance is that it measures only sting interference and the model aerodynamics forces. There are no struts and wires attached to the model.

References 1, 2 and 3 present a detailed discussion of the calibration of the internal balance and associated data reduction procedures. They all use a linear approximation method to determine the interactions between the various components of the balance. By using linear approximation, satisfactory results for low values of forces and moments can be obtained, such as subsonic wind tunnel data. Because the nonlinear interactions in the subsonic regime are small, the nonlinear interactions can be neglected.

The interactions between the various components of the
balance are basically nonlinear. They are caused essentially by structural deflections. For this project, wind tunnel data are obtained in the transonic regime and the forces and moments are greater than those obtained in the subsonic regime. By checking data in reference 4, we know that satisfactory results can not be obtained by using linear approximation. Thus, calibration procedures that account for the nonlinear interactions are required.

The balance which is used in this project is a 5-component strain-gage balance. It was developed and calibrated for acquiring transonic wind tunnel data. A Fast Fourier Transform code was developed to filter the oscillations in the raw balance data as part of the data reduction procedure. This thesis presents the calibration procedure of the balance, the filter theory, and balance data for two representative helicopter rotor blade sections at low transonic Mach numbers.
CHAPTER II
FACILITY DESCRIPTION

This chapter describes the components of the load measurement system that were used. These components included the 5-component strain-gage balance, a data acquisition system and models. In addition, the wind tunnel will be described.

2.1 Wind Tunnel and Data Acquisition System
2.1.1 Wind Tunnel

The facility used in this project was the University of Texas at Arlington High-Reynolds Number Transonic Wind Tunnel (HIRT), which is a Ludwig tube tunnel with exceptional flow quality and Reynolds number simulation capability. The basic tunnel utilizes many of the components from the original Pilot HIRT facility developed at the U.S. Air Force Arnold Engineering Development Center in the early 1970's as a prototype of the proposed Air Force concept for the National Transonic Facility. The wind tunnel was donated to UTA in 1978. The components of the tunnel (Fig. 2.1) consist of a charge tube, a convergent nozzle, a test section, an ejector flap section, a diffuser and a starting valve.
2.1.2 Theory of Operation

Ludwig tube tunnels are based on an unsteady expansion wave concept for acceleration of high pressure air stored within a cylindrical charge tube to transonic Mach number. The flow process basically resembles the flow within the driver tube of a conventional shock tube. The tunnel flow is initiated by charging the entire system to 75 ~ 750 psia, and rapidly opening the starting valve. This action generates an unsteady expansion wave that propagates upstream through the diffuser, test section, nozzle and into the charge tube to initiate the flow towards the open valve at the downstream end of the tunnel. Once the expansion wave clears the nozzle, steady flow within the test section is maintained for the time duration required for the expansion wave to travel the length of the charge tube, reflect and return to the test section. The 111 foot length of the charge tube provides a theoretical steady flow period of 185 msec. However, this is reduced to about 100 msec by the time actually required for opening the starting valve and by unsteady phenomena associated with the flow initiation in the plenum cavity surrounding the porous wall test section.

A typical total pressure diagram is shown in Fig. 2.2. From Fig. 2.2, we can see that there are two steady state periods during the run time. The first steady state period
Fig. 2.2 Total pressure diagram
occurs between 130 msec to 220 msec. A second near-steady state period occurs between 350 msec to 440 msec. The second period is generated by re-reflection of the initial expansion wave from the starting valve (Fig. 2.3). Although the total pressure is not constant during this period, the pressure drop is gradual and satisfactory wind tunnel data can be obtained. Because the test section static pressures during the second near-steady state period are about half of those during the first steady state, the Reynolds number is about half of that of the first steady state period. So, it is possible to obtain two sets of data for different Reynolds numbers during the same run. The Mach number for the second near-steady state period is slightly lower than that for the first steady state period. The data for the second near-steady state period is obtained about 40 msec before the end of this period. (Note: a more detailed discussion of the UTA transonic wind tunnel operation is presented in references 5 and 6).

2.1.3 Data Acquisition and Facility Control Computer

Test facility operational control, data acquisition and data processing are accomplished by a custom-designed microcomputer system utilizing a PSP Data Acquisition Software Package. The control computer, which is located adjacent to the test section of the tunnel, controls the
Fig. 2.3 X-t diagram

reflected waves

incident waves

first steady state period
operation of the wind tunnel by sending electrical signals to solenoid valves to control opening the sliding sleeve valve to initiate the tunnel flow, cutting the diaphragm to initiate the plenum exhaust system flow and adjustment of a Flex-Flo valve used in the tunnel starting sequence. The sliding sleeve valve is then closed after termination of the run.

During the run, the raw wind tunnel data are stored into the control computer memory. After the completion of the tunnel run, the data are then transferred to the main computer located inside the control room via an IEEE 488 bus and processed while the tunnel is recharged for the next run. The data can be displayed on the computer CRT display screen, or printed out on a dot-matrix printer. A 5$\frac{1}{4}$ inch floppy disk system is available for permanent storage of the data.

2.2 The Balance

The balance (Fig. 2.4) is a 5-component, internal strain-gage load measuring instrument with maximum design loads of 500 lb normal force, 75 lb chord force, 1000 lb-in rolling moment, 150 lb-in yawing moment, and 140 lb-in pitching moment. Its size and load capacity make it suitable for use in the UTA transonic wind tunnel. This balance is mounted to an access port on the right hand side of the test
Note: not to scale

Fig. 2.4 Balance configuration (continued)
Fig. 2.4 Balance configuration (concluded)
section. So, it is a side-wall balance.

A calibration body (Fig. 2.5) is mounted over the balance by screws. The calibration body is notched at one inch intervals. So, various moments can be obtained by changing the location of a constant dead weight on the calibration body. Use of the calibration body will be discussed in Chapter III. A complete description of the balance configuration is presented in reference 4. Please refer to it for further information about the balance configuration.

2.3 The Model

Two different semi-span rotor blade models were used for this project to provide proof-of-concept data for evaluation of the balance. One employs an exact NACA 0012 airfoil section (Fig. 2.6). The other uses an approximate representation of the NACA 0012 airfoil (Fig. 2.7) and was used as a vortex generator in an earlier test program (Ref. 6). Let us define the first model as model A and the second model as model B. They are mounted to the balance by screws. A model change can be accomplished within about forty minutes by two persons. This includes removal of the diffuser section from the wind tunnel for access to the test section. The installation of the model with the balance in the transonic
Fig. 2.5 Calibration body

Note: all dimensions are in inches
Fig. 2.6 Model A

Fig. 2.7 Model B
wind tunnel is presented in Fig. 2.8. The dimensions of the models are shown in Table 1.

<table>
<thead>
<tr>
<th></th>
<th>MODEL A</th>
<th>MODEL B</th>
</tr>
</thead>
<tbody>
<tr>
<td>SPAN (in)</td>
<td>4(\frac{5}{16})</td>
<td>5.1</td>
</tr>
<tr>
<td>CHORD (in)</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>ADAPTOR (in)</td>
<td>0.8125</td>
<td>0.46875</td>
</tr>
</tbody>
</table>
Fig. 2.8 Profile of installation of the side wall balance and model in the wind tunnel
CHAPTER III

CALIBRATION

The balance was originally calibrated by Modern Machine & Tool Company in April 1989. The complete calibration procedure and data are presented in reference 4.

After receiving the balance, a check calibration was performed to verify the calibration data supplied by Modern Machine & Tool Company. After a period of use, channel 3 of the balance was found to be out of order. The balance was sent back to Modern Machine and Tool Company for repair. After the balance was returned, a complete calibration for channel 3 was conducted in January 1991. In this chapter, the data obtained from the calibration performed in January 1991 are compared with those data presented in reference 4.

3.1 Calibration Data

Before using the balance, a check on the electrical zeros and bridge resistances should be performed and all the data should be recorded. After a test is finished, the same check should be performed again to compare the post run data with that obtained before using the balance. If both sets of data are nearly the same, then the balance can be considered
to be in good condition. The following tables present a comparsion between the original calibration data (Jan. 29, 1991) and the re-calibration data (May 22, 1991).

3.1.1 Electrical zero (balance zero reading without the calibration body)

Balance supply voltage : 5 VDC

<table>
<thead>
<tr>
<th></th>
<th>R1</th>
<th>R2</th>
<th>R3 (CF)</th>
<th>R4 (MZ)</th>
<th>R5 (PM)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jan 29, 1991</td>
<td>0.237</td>
<td>0.149</td>
<td>1.053</td>
<td>0.560</td>
<td>0.410</td>
</tr>
<tr>
<td>May 22, 1991</td>
<td>0.222</td>
<td>0.134</td>
<td>1.061</td>
<td>0.546</td>
<td>0.413</td>
</tr>
</tbody>
</table>

Note: (1) all measurements are in mv.
(2) (R2-R1) corresponds to normal force (NF).
(R2+R1) corresponds to root bending moment (MX).

3.1.2 Resistor Calibration Check

<table>
<thead>
<tr>
<th></th>
<th>R1</th>
<th>R2</th>
<th>R3</th>
<th>R4</th>
<th>R5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bridge Resistance (ohms)</td>
<td>350</td>
<td>350</td>
<td>350</td>
<td>350</td>
<td>700</td>
</tr>
<tr>
<td>R-cal Resistor (ohms)</td>
<td>69.7K</td>
<td>69.7K</td>
<td>69.7K</td>
<td>199.6K</td>
<td>169.7K</td>
</tr>
<tr>
<td>May 22, 1991 R-cal output (mv)</td>
<td>6.174</td>
<td>6.090</td>
<td>7.025</td>
<td>2.626</td>
<td>5.577</td>
</tr>
</tbody>
</table>
3.1.3 Bridge Resistance Check

A = +POWER
B = -SIGNAL
C = -POWER
D = +SIGNAL

Date: March 3, 1989

<table>
<thead>
<tr>
<th></th>
<th>R1</th>
<th>R2</th>
<th>R3</th>
<th>R4</th>
<th>R5</th>
</tr>
</thead>
<tbody>
<tr>
<td>A-C</td>
<td>374</td>
<td>374</td>
<td>374</td>
<td>374</td>
<td>745</td>
</tr>
<tr>
<td>B-D</td>
<td>352</td>
<td>352</td>
<td>352</td>
<td>352</td>
<td>702</td>
</tr>
<tr>
<td>A-B</td>
<td>286</td>
<td>287</td>
<td>286</td>
<td>287</td>
<td>549</td>
</tr>
<tr>
<td>B-C</td>
<td>265</td>
<td>265</td>
<td>265</td>
<td>265</td>
<td>549</td>
</tr>
<tr>
<td>C-D</td>
<td>265</td>
<td>265</td>
<td>265</td>
<td>265</td>
<td>549</td>
</tr>
<tr>
<td>D-A</td>
<td>286</td>
<td>286</td>
<td>286</td>
<td>286</td>
<td>549</td>
</tr>
</tbody>
</table>

Date: May 22, 1991

<table>
<thead>
<tr>
<th></th>
<th>R1</th>
<th>R2</th>
<th>R3</th>
<th>R4</th>
<th>R5</th>
</tr>
</thead>
<tbody>
<tr>
<td>A-C</td>
<td>374.2</td>
<td>374.4</td>
<td>374.4</td>
<td>374.7</td>
<td>745.8</td>
</tr>
<tr>
<td>B-D</td>
<td>353.1</td>
<td>353.5</td>
<td>353.7</td>
<td>353.5</td>
<td>703.2</td>
</tr>
<tr>
<td>A-B</td>
<td>286.8</td>
<td>287.2</td>
<td>286.7</td>
<td>287.3</td>
<td>549.5</td>
</tr>
<tr>
<td>B-C</td>
<td>265.8</td>
<td>266.0</td>
<td>266.1</td>
<td>265.9</td>
<td>549.7</td>
</tr>
<tr>
<td>C-D</td>
<td>265.7</td>
<td>266.0</td>
<td>266.2</td>
<td>265.8</td>
<td>549.5</td>
</tr>
<tr>
<td>D-A</td>
<td>286.7</td>
<td>287.0</td>
<td>286.7</td>
<td>287.1</td>
<td>549.4</td>
</tr>
</tbody>
</table>

Note: all measurements are in ohms.

3.2 Data Reduction Equation

Because the interactions between the various components of the balance are nonlinear, we adopt an iterated equation
to determine the interactions.

The equation (Ref. 4) is

\[ F^{i+1} = F^0 - M \cdot A^T \]  \hspace{1cm} \text{(3.1)}

\begin{equation}
F^0 = \begin{bmatrix}
F^0_0 \\
F^0_1 \\
F^0_2 \\
F^0_3 \\
F^0_4 \\
\end{bmatrix}
\end{equation}

where \( F^0 \) is a 5x1 matrix.

\[ A = \begin{bmatrix}
F^1_0 & F^1_0 & F^1_1 & F^1_1 & F^1_1 & F^1_1 & F^1_1 & F^1_1 & F^1_1 & F^1_1 \\
F^1_0 & F^1_0 & F^1_1 & F^1_1 & F^1_1 & F^1_1 & F^1_1 & F^1_1 & F^1_1 & F^1_1 \\
F^1_0 & F^1_1 & F^1_2 & F^1_2 & F^1_2 & F^1_2 & F^1_2 & F^1_2 & F^1_2 & F^1_2 \\
F^1_1 & F^1_0 & F^1_2 & F^1_2 & F^1_2 & F^1_2 & F^1_2 & F^1_2 & F^1_2 & F^1_2 \\
F^1_1 & F^1_1 & F^1_2 & F^1_2 & F^1_2 & F^1_2 & F^1_2 & F^1_2 & F^1_2 & F^1_2 \\
F^1_1 & F^1_1 & F^1_2 & F^1_2 & F^1_2 & F^1_2 & F^1_2 & F^1_2 & F^1_2 & F^1_2 \\
\end{bmatrix}
\]

\( F^0_0 \) is the uncorrected normal force (NF).
\( F^0_1 \) is the uncorrected root bending moment (MX).
\( F^0_2 \) is the uncorrected chord force (CF).
\( F^0_3 \) is the uncorrected yawing moment (MZ).
\( F^0_4 \) is the uncorrected pitching moment (PM).
\( M \), balance matrix, is a 5x13 matrix.
The complete description of this equation, the balance matrix elements and the determination of $F^{<0>}$ are presented in Appendix I.

3.3 Determination of the Interaction Coefficients

The determination of the balance matrix elements (interaction coefficients) is outlined below. The calibration data obtained on January 29, 1991 are used as an example.

3.3.1 Raw Balance Data

The calibration body (Fig. 2.5) was assembled to the balance by screws. The hanger was mounted on the calibration body so that dead weights can be put on the balance to obtain calibration data. By moving the location of the hanger on the calibration body, different moments can be obtained.

By using the calibration body and hanger, the following raw calibration data for R3 can be obtained (Note: check Fig. 2.5 for the locations of A, A1, B, ..., E, E1).
<table>
<thead>
<tr>
<th>Point A on +NF plane</th>
<th>Point B on +NF plane</th>
</tr>
</thead>
<tbody>
<tr>
<td>Loading (lb)</td>
<td>Reading (mv)</td>
</tr>
<tr>
<td>Calibration</td>
<td>1.100</td>
</tr>
<tr>
<td>body &amp; hanger</td>
<td></td>
</tr>
<tr>
<td>50</td>
<td>1.118</td>
</tr>
<tr>
<td>100</td>
<td>1.138</td>
</tr>
<tr>
<td>150</td>
<td>1.156</td>
</tr>
<tr>
<td>200</td>
<td>1.175</td>
</tr>
<tr>
<td>150</td>
<td>1.157</td>
</tr>
<tr>
<td>100</td>
<td>1.138</td>
</tr>
<tr>
<td>50</td>
<td>1.119</td>
</tr>
<tr>
<td>Calibration</td>
<td>1.100</td>
</tr>
<tr>
<td>body &amp; hanger</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Point C on +NF plane</th>
<th>Point A1 on -NF plane</th>
</tr>
</thead>
<tbody>
<tr>
<td>Loading (lb)</td>
<td>Reading (mv)</td>
</tr>
<tr>
<td>Calibration</td>
<td>1.114</td>
</tr>
<tr>
<td>body &amp; hanger</td>
<td></td>
</tr>
<tr>
<td>50</td>
<td>1.153</td>
</tr>
<tr>
<td>100</td>
<td>1.192</td>
</tr>
<tr>
<td>150</td>
<td>1.231</td>
</tr>
<tr>
<td>200</td>
<td>1.270</td>
</tr>
<tr>
<td>150</td>
<td>1.231</td>
</tr>
<tr>
<td>100</td>
<td>1.192</td>
</tr>
<tr>
<td>50</td>
<td>1.153</td>
</tr>
<tr>
<td>Calibration</td>
<td>1.114</td>
</tr>
<tr>
<td>body &amp; hanger</td>
<td></td>
</tr>
<tr>
<td>Point B1 on -NF plane</td>
<td>Point C1 on -NF plane</td>
</tr>
<tr>
<td>----------------------</td>
<td>----------------------</td>
</tr>
<tr>
<td><strong>Loading (lb)</strong></td>
<td><strong>Reading (mv)</strong></td>
</tr>
<tr>
<td>Calibration body &amp; hanger</td>
<td>1.048</td>
</tr>
<tr>
<td>50</td>
<td>1.021</td>
</tr>
<tr>
<td>100</td>
<td>0.995</td>
</tr>
<tr>
<td>150</td>
<td>0.968</td>
</tr>
<tr>
<td>200</td>
<td>0.941</td>
</tr>
<tr>
<td>150</td>
<td>0.967</td>
</tr>
<tr>
<td>100</td>
<td>0.994</td>
</tr>
<tr>
<td>50</td>
<td>1.021</td>
</tr>
<tr>
<td>Calibration body &amp; hanger</td>
<td>1.048</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Point D on +NF plane</th>
<th>Point E on +NF plane</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Loading (lb-in)</strong></td>
<td><strong>Reading (mv)</strong></td>
</tr>
<tr>
<td>0</td>
<td>1.094</td>
</tr>
<tr>
<td>70</td>
<td>1.119</td>
</tr>
<tr>
<td>140</td>
<td>1.145</td>
</tr>
<tr>
<td>70</td>
<td>1.119</td>
</tr>
<tr>
<td>0</td>
<td>1.095</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Point D1 on -NF plane</th>
<th>Point E1 on -NF plane</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Loading (lb-in)</strong></td>
<td><strong>Reading (mv)</strong></td>
</tr>
<tr>
<td>0</td>
<td>1.071</td>
</tr>
<tr>
<td>-70</td>
<td>1.051</td>
</tr>
<tr>
<td>-140</td>
<td>1.031</td>
</tr>
<tr>
<td>-70</td>
<td>1.051</td>
</tr>
<tr>
<td>0</td>
<td>1.071</td>
</tr>
<tr>
<td>Point A on +CF plane</td>
<td>Point B on +CF plane</td>
</tr>
<tr>
<td>---------------------</td>
<td>---------------------</td>
</tr>
<tr>
<td><strong>Loading (lb)</strong></td>
<td><strong>Reading (mv)</strong></td>
</tr>
<tr>
<td>Calibration body &amp; hanger</td>
<td>5.271</td>
</tr>
<tr>
<td>10</td>
<td>6.173</td>
</tr>
<tr>
<td>20</td>
<td>7.080</td>
</tr>
<tr>
<td>30</td>
<td>7.982</td>
</tr>
<tr>
<td>20</td>
<td>7.086</td>
</tr>
<tr>
<td>10</td>
<td>6.187</td>
</tr>
<tr>
<td>Calibration body &amp; hanger</td>
<td>5.287</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Point C on +CF plane</th>
<th>Point A1 on -CF plane</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Loading (lb)</strong></td>
<td><strong>Reading (mv)</strong></td>
</tr>
<tr>
<td>Calibration body &amp; hanger</td>
<td>5.270</td>
</tr>
<tr>
<td>10</td>
<td>6.173</td>
</tr>
<tr>
<td>20</td>
<td>7.078</td>
</tr>
<tr>
<td>30</td>
<td>7.981</td>
</tr>
<tr>
<td>20</td>
<td>7.086</td>
</tr>
<tr>
<td>10</td>
<td>6.185</td>
</tr>
<tr>
<td>Calibration body &amp; hanger</td>
<td>5.284</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Point B1 on -CF plane</th>
<th>Point C1 on -CF plane</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Loading (lb)</strong></td>
<td><strong>Reading (mv)</strong></td>
</tr>
<tr>
<td>Calibration body &amp; hanger</td>
<td>-3.112</td>
</tr>
<tr>
<td>10</td>
<td>-4.012</td>
</tr>
<tr>
<td>20</td>
<td>-4.915</td>
</tr>
<tr>
<td>30</td>
<td>-5.817</td>
</tr>
<tr>
<td>20</td>
<td>-4.922</td>
</tr>
<tr>
<td>10</td>
<td>-4.023</td>
</tr>
<tr>
<td>Calibration body &amp; hanger</td>
<td>-3.123</td>
</tr>
</tbody>
</table>
3.3.2 Reduced Balance Data

Definition:

\[ \text{Delta} = \text{reading of loading} - \text{reading of calibration body} & \text{ hanger} \]

<table>
<thead>
<tr>
<th>Loading point</th>
<th>Loading</th>
<th>Delta</th>
</tr>
</thead>
<tbody>
<tr>
<td>A on +NF plane</td>
<td>200 (lb)</td>
<td>0.075</td>
</tr>
<tr>
<td>B on +NF plane</td>
<td>200 (lb)</td>
<td>0.118</td>
</tr>
<tr>
<td>C on +NF plane</td>
<td>200 (lb)</td>
<td>0.156</td>
</tr>
<tr>
<td>A1 on -NF plane</td>
<td>200 (lb)</td>
<td>-0.062</td>
</tr>
<tr>
<td>B1 on -NF plane</td>
<td>200 (lb)</td>
<td>-0.107</td>
</tr>
<tr>
<td>C1 on -NF plane</td>
<td>200 (lb)</td>
<td>-0.147</td>
</tr>
<tr>
<td>D on +NF plane</td>
<td>140 (lb-in)</td>
<td>0.051</td>
</tr>
<tr>
<td>E on +NF plane</td>
<td>-140 (lb-in)</td>
<td>-0.051</td>
</tr>
<tr>
<td>D1 on -NF plane</td>
<td>-140 (lb-in)</td>
<td>-0.040</td>
</tr>
<tr>
<td>E1 on -NF plane</td>
<td>140 (lb-in)</td>
<td>0.041</td>
</tr>
<tr>
<td>A on +CF plane</td>
<td>30 (lb)</td>
<td>2.711</td>
</tr>
<tr>
<td>B on +CF plane</td>
<td>30 (lb)</td>
<td>2.706</td>
</tr>
<tr>
<td>C on +CF plane</td>
<td>30 (lb)</td>
<td>2.711</td>
</tr>
<tr>
<td>A1 on -CF plane</td>
<td>30 (lb)</td>
<td>-2.708</td>
</tr>
<tr>
<td>B1 on -CF plane</td>
<td>30 (lb)</td>
<td>-2.705</td>
</tr>
<tr>
<td>C1 on -CF plane</td>
<td>30 (lb)</td>
<td>-2.712</td>
</tr>
</tbody>
</table>

3.3.3 Interaction Coefficients

Definition:

\[ A, A1, B, \ldots, E, E1 \] correspond to the Delta of loading points \[ A, A1, B, \ldots, E, E1 \] in section 3.2.2.

There are certain nonlinear relations between the
outputs (mv) of the balance and the corresponding forces or moments acting on the balance. Those nonlinear relations are outlined below.

**Loading points on +NF & -NF plane**

\[ A = NF + NF^2 \]
\[ A_1 = -NF + NF^2 \]
\[ B = NF + NF^2 + \frac{MX}{2} + \frac{MX^2}{4} + \frac{NF \times MX}{2} \]
\[ B_1 = -NF + NF^2 - \frac{MX}{2} + \frac{MX^2}{4} + \frac{NF \times MX}{2} \]
\[ C = NF + NF^2 + MX + MX^2 + NF \times MX \quad \text{(3.2)} \]
\[ C_1 = -NF + NF^2 - MX + MX^2 + NF \times MX \]
\[ D = PM + PM^2 + \frac{NF \times PM}{25} \]
\[ E = -PM + PM^2 - \frac{NF \times PM}{25} \]
\[ D_1 = -PM + PM^2 + \frac{NF \times PM}{25} \]
\[ E_1 = PM + PM^2 - \frac{NF \times PM}{25} \]

From the above relations, we can obtain the following interaction coefficients

\[ NF = \frac{(A - A_1)}{2} \]
\[ NF^2 = \frac{(A + A_1)}{2} \]
\[ MX = (C - C_1) - (B - B_1) \]
\[ MX^2 = (C + C_1) - 2 \times (B + B_1) + (A + A_1) \quad \text{(3.3)} \]
\[ NF \times MX = 2 \times (B + B_1) - 0.5 \times (C + C_1) - 1.5 \times (A + A_1) \]
\[ PM = \frac{((D - D_1) - (E - E_1))}{4} \]
\[ PM^2 = \frac{((D + D_1) + (E + E_1))}{4} \]
NF * PM = ((D - E) + (D1 - E1)) * 6.25

**Loading point on +CF & -CF plane**

\[
A = CF + CF^2 \\
A1 = -CF + CF^2 \\
B = CF + CF^2 + \frac{MZ}{2} + \frac{MZ^2}{4} + \frac{CF * MZ}{2} \\
B1 = -CF + CF^2 - \frac{MZ}{2} + \frac{MZ^2}{4} + \frac{CF * MZ}{2} \\
C = CF + CF^2 + MZ + MZ^2 + CF * MZ \\
C1 = -CF + CF^2 - MZ + MZ^2 + CF * MZ
\]

From the above relations, we can obtain the following interaction coefficients

\[
CF = \frac{(A - A1)}{2} \\
CF^2 = \frac{(A + A1)}{2} \\
MZ = (C - C1) - (B - B1) \\
MZ^2 = (C + C1) - 2 * (B + B1) + (A + A1) \\
CF * MZ = 2 * (B + B1) - 0.5 * (C + C1) - 1.5 * (A + A1)
\]

By using the above equations and data shown in section 3.3.2, the interaction coefficients for R3 (chord force) can be obtained.

<table>
<thead>
<tr>
<th>NF</th>
<th>NF^2</th>
<th>MX</th>
<th>MX^2</th>
<th>NF * MX</th>
</tr>
</thead>
<tbody>
<tr>
<td>CF</td>
<td>0.0685</td>
<td>6.5*10^{-3}</td>
<td>0.078</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>PM</td>
<td>PM^2</td>
<td>NF * PM</td>
<td>CF</td>
</tr>
<tr>
<td>------</td>
<td>---------</td>
<td>--------</td>
<td>----------</td>
<td>--------</td>
</tr>
<tr>
<td>CF</td>
<td>0.04575</td>
<td>5*10^-4</td>
<td>0.13125</td>
<td>2.7095</td>
</tr>
<tr>
<td></td>
<td>MZ</td>
<td>MZ^2</td>
<td>CF * MZ</td>
<td></td>
</tr>
<tr>
<td>CF</td>
<td>0.012</td>
<td>0</td>
<td>-2*10^-3</td>
<td></td>
</tr>
</tbody>
</table>

By using the same procedure, all the interaction coefficients of NF, MX, MZ, PM can be obtained from reference 4. They are presented in Table 2.

3.3.4 Reduced Interaction Coefficients

The interaction coefficients in Table 2 should be further reduced so that they can be used in the balance matrix (M). The interaction coefficients for chord force (column 3) are shown as an example. The outline of the reduction procedure is presented as following (Note: the sensitivity constants for NF, MX, CF, MZ, PM are presented in Appendix I).

1. Multiply all the coefficients in column 3 by the CF sensitivity constant (11.063526).
2. All the coefficients in column 3 divide their corresponding row (force or moment), respectively. For example:
Table 2 Interaction Coefficients

<table>
<thead>
<tr>
<th></th>
<th>NF</th>
<th>MX</th>
<th>CF</th>
<th>MZ</th>
<th>PM</th>
</tr>
</thead>
<tbody>
<tr>
<td>NF</td>
<td>0.3155</td>
<td>2.4925</td>
<td>0.0685</td>
<td>-8.5\times10^{-3}</td>
<td>-0.0285</td>
</tr>
<tr>
<td>NF^2</td>
<td>5.0\times10^{-4}</td>
<td>-2.5\times10^{-3}</td>
<td>6.5\times10^{-3}</td>
<td>-5.0\times10^{-4}</td>
<td>0.0155</td>
</tr>
<tr>
<td>MX</td>
<td>-0.062</td>
<td>2.702</td>
<td>0.078</td>
<td>-0.012</td>
<td>-0.05</td>
</tr>
<tr>
<td>MX^2</td>
<td>4.0\times10^{-3}</td>
<td>-0.016</td>
<td>0</td>
<td>-2.0\times10^{-3}</td>
<td>2.0\times10^{-3}</td>
</tr>
<tr>
<td>NF \times MX</td>
<td>-4.0\times10^{-3}</td>
<td>0.012</td>
<td>-2.0\times10^{-3}</td>
<td>3.0\times10^{-3}</td>
<td>-0.011</td>
</tr>
<tr>
<td>PM</td>
<td>-8.3\times10^{-3}</td>
<td>8.25\times10^{-3}</td>
<td>0.04575</td>
<td>-0.02275</td>
<td>5.6255</td>
</tr>
<tr>
<td>PM^2</td>
<td>-7.5\times10^{-4}</td>
<td>2.75\times10^{-3}</td>
<td>5.0\times10^{-4}</td>
<td>2.5\times10^{-4}</td>
<td>-1.0\times10^{-3}</td>
</tr>
<tr>
<td>NF \times PM</td>
<td>6.25\times10^{-3}</td>
<td>-0.03125</td>
<td>0.013125</td>
<td>0.01875</td>
<td>-0.0875</td>
</tr>
<tr>
<td>CF</td>
<td>-9.5\times10^{-3}</td>
<td>-0.0205</td>
<td>2.7095</td>
<td>0.4155</td>
<td>9.5\times10^{-3}</td>
</tr>
<tr>
<td>CF^2</td>
<td>5.0\times10^{-4}</td>
<td>1.5\times10^{-3}</td>
<td>1.5\times10^{-3}</td>
<td>-5.0\times10^{-4}</td>
<td>5.0\times10^{-4}</td>
</tr>
<tr>
<td>MZ</td>
<td>0</td>
<td>-4.0\times10^{-3}</td>
<td>0.012</td>
<td>0.434</td>
<td>-2.0\times10^{-3}</td>
</tr>
<tr>
<td>MZ^2</td>
<td>-2.0\times10^{-3}</td>
<td>-2.0\times10^{-3}</td>
<td>0</td>
<td>-1.0\times10^{-3}</td>
<td>-3.0\times10^{-3}</td>
</tr>
<tr>
<td>CF \times MZ</td>
<td>1.0\times10^{-3}</td>
<td>1.0\times10^{-3}</td>
<td>-2.0\times10^{-3}</td>
<td>1.5\times10^{-3}</td>
<td>6.5\times10^{-3}</td>
</tr>
</tbody>
</table>

-2\times10^{-3} corresponding to row NF \times MX in column 3. should be divided by \((0.3155\times630.51702)\times(2.702\times147.95088)\). The reasons are

(i) 0.3155 is the interaction coefficient corresponding to column NF and row NF in Table 2. 630.51702 is the sensitivity constant for normal force (NF). So, 0.3155\times630.51702 represents the influence of NF.

(ii) 2.702 is the interaction coefficient corresponding to column MX and row MX in Table 2. 147.95088 is the sensitivity constant for root bending moment (MX). So, 2.702\times147.95088 represents the influence of MX.

By using the above procedure, the reduced interaction
coefficients for chord force (CF) can be obtained, and are shown in the table below.

<table>
<thead>
<tr>
<th></th>
<th>CF</th>
</tr>
</thead>
<tbody>
<tr>
<td>NF</td>
<td>3.8096*10^{-3}</td>
</tr>
<tr>
<td>NF^2</td>
<td>1.8172*10^{-6}</td>
</tr>
<tr>
<td>MX</td>
<td>2.1586*10^{-3}</td>
</tr>
<tr>
<td>MX^2</td>
<td>0</td>
</tr>
<tr>
<td>NF * MX</td>
<td>-2.7824*10^{-7}</td>
</tr>
<tr>
<td>PM</td>
<td>3.6157*10^{-3}</td>
</tr>
<tr>
<td>PM^2</td>
<td>2.8228*10^{-7}</td>
</tr>
<tr>
<td>NF * PM</td>
<td>5.2114*10^{-5}</td>
</tr>
<tr>
<td>CF</td>
<td>1</td>
</tr>
<tr>
<td>CF^2</td>
<td>1.8468*10^{-5}</td>
</tr>
<tr>
<td>MZ</td>
<td>2.2208*10^{-3}</td>
</tr>
<tr>
<td>MZ^2</td>
<td>0</td>
</tr>
<tr>
<td>CF * MZ</td>
<td>-1.2347*10^{-5}</td>
</tr>
</tbody>
</table>

The complete reduced interaction coefficients and sensitivity constants are presented in Appendix I.
CHAPTER IV
FILTER

Because there are a lot of oscillations in the raw balance data (Fig. 4.1), a digital filter code is used to filter the high frequency oscillations in order to obtain the steady state values (Fig. 4.2). A Fast Fourier Transform (FFT) theorem was used to develop the filter code (Appendix II). The outline of the FFT theorem and the computer algorithm will be presented below (Ref. 7).

4.1 Outline of Fast Fourier Transform
4.1.1 Continuous Fourier Representation

Let \( f \) be the function we want to approximate by the FFT. Assume \( f \) be a continuous function in \( 0 \leq t \leq T \) except possibly for a finite number of points. The Fourier series representation for \( f \) in \( [0,T] \) is given by

\[
f(t) = \sum_{k=-\infty}^{\infty} F_k \exp(2\pi ikt/T) \quad \text{------------------------ (4.1)}
\]

where the \( l \)th Fourier coefficient \( F_1 \) is given by

\[
F_1 = \frac{1}{T} \int_0^T f(t) \exp(-2\pi i lt/T) dt \quad \text{------------------------ (4.2)}
\]
Fig. 4.1 Raw balance data

Fig. 4.2 Filtered balance data
4.1.2 Discrete Fourier Representation

Given \( N \) numbers \( \{f_j\}_{j=0}^{N-1} \), the associated discrete Fourier coefficients \( \{F_k^{(N)}\}_{k=0}^{N-1} \) are defined by

\[
F_k^{(N)} = \frac{1}{N} \sum_{j=0}^{N-1} f_j \exp(-2\pi ijk/N) \tag{4.3}
\]

The relationship between \( \{f_j\} \) and \( \{F_k^{(N)}\} \) is determined by the following formula:

\[
f_j = \sum_{k=0}^{N-1} F_k^{(N)} \exp(2\pi ijk/N) \tag{4.4}
\]

Corollary:

Let the data

\[
f_j = f(jT/N) \quad j = 0, 1, 2, \ldots, N-1. \tag{4.5}
\]

be given. Then the trigonometric polynomial equation (4.4) interpolates these data.

\[
f_k^{(N)}(jT/N) = f_j = f(jT/N) \quad j = 0, 1, 2, \ldots, N-1.
\]

provided \( \{F_k^{(N)}\}_{k=0}^{N-1} \) is the set of discrete Fourier coefficients defined by (4.3).
4.1.3 Accuracy of Discrete Fourier Coefficients

Let \( \{F_k\}_{k=0}^{\infty} \) be defined by (4.2) and let \( \{F_k^{(N)}\}_{k=0}^{N-1} \) be defined by (4.3) with \( f_j \) given by (4.5). Then

\[
F_k^{(N)} = F_k + \sum_{l \neq 0} F_{k+lN} \quad \text{----------------------------- (4.6)}
\]

The second term \( \sum_{l \neq 0} F_{k+lN} \) on the right hand side of (4.12) is called the aliasing error. Aliasing occurs when high frequency information in the exact representation cascades down into the lower frequencies of the discrete representation. Since

\[
F_{k+lN} \longrightarrow 0 \quad \text{as} \quad N \longrightarrow \infty \quad \text{----------------------------- (4.7)}
\]

Proof:

Let \( S = k+lN \)

So, \( S \longrightarrow \infty \) as \( N \longrightarrow \infty \)

From (4.2),

\[
F_s = \frac{1}{T} \int_0^T f(t) \left( \cos \frac{2\pi st}{T} - i \sin \frac{2\pi st}{T} \right) dt
\]

\[
F_s = \frac{1}{T} \int_0^T f(t) \cos \frac{2\pi st}{T} \, dt - i \frac{1}{T} \int_0^T f(t) \sin \frac{2\pi st}{T} \, dt
\]

\[
F_s = \frac{1}{T} \ast \frac{T}{2\pi s} \int_0^T f(t) \, d\left( \sin \frac{2\pi st}{T} \right) + i \frac{1}{T} \ast \frac{T}{2\pi s} \int_0^T f(t) \, d\left( \cos \frac{2\pi st}{T} \right)
\]

\[
F_s = \frac{1}{2\pi s} \left( \int_0^T f(t) \, d\left( \sin \frac{2\pi st}{T} \right) + i \int_0^T f(t) \, d\left( \cos \frac{2\pi st}{T} \right) \right) \quad \text{----------------------------- (4.8)}
\]
But, \( \frac{1}{2\pi S} \rightarrow 0 \) as \( S \rightarrow \infty \)

So, (4.8) becomes

\[ F_{k+1N} = F_c = 0 \]

So, (4.7) is valid.

From (4.7), we know that aliasing is diminished by increasing the number of samples. So, \( F^{(N)}_k \) will approach \( F_k \) when the number of samples is increased.

Conclusion:

Only half of the discrete Fourier coefficients (4.3) give reasonable approximation to their continuous counterparts. The others have an unacceptable level of reverse aliasing error. That is, for \( k > \frac{N}{2} \), \( F^{(N)}_k \) is influenced by some \( F_j \) for \( j < \frac{N}{2} \). (Recall, \( F^{(N)}_k = \sum_{r=-\infty}^{\infty} F_{k+rN} \)).

In using the Fast Fourier Transform, we need to avoid the unacceptable reverse aliasing error. So, we use

\[
F^{(N)}_j = F^{(N)}_0 + \sum_{k=1}^{[N/2]-1} \left\{ F^{(N)}_k \exp(2\pi i jk/N) + F^{(N)}_{-k} \exp(-2\pi i jk/N) \right\}
\]

\[ \text{-------- (4.9)} \]

instead of (4.4) to approximate \( f \).
4.2 Computational Algorithm

Assumptions:

(1) N is a power of 2\(^t\), where \(t\) is an integer. (for example we have 1024 samples per channel, thus, \(N = 2^{10}\)).

(2) \(W_N^k = \exp(2\pi ik/N)\)

(3) \(F_k^{(N)}[W_N^k \mid f_0, f_1, f_2, \ldots, f_{N-1}]\) denotes the following trigonometric polynomial in \(W\), i.e.

\[
F_k^{(N)}[W_N^k \mid f_0, f_1, \ldots, f_{N-1}]
= f_0 + f_1 W_N^k + f_2 (W_N^k)^2 + \ldots + f_{N-1} (W_N^k)^{N-1}
= \sum_{j=0}^{N-1} f_j (W_N^k)^j
\]

where \(f_j\) is the sample data.

Our task is to evaluate

\[
F_k^{(N)} = \sum_{j=0}^{N-1} f_j (W_N^k)^j
\]

For example:

for \(N = 2\), we need to evaluate \(F_0^{(2)}\) and \(F_1^{(2)}\).

\[
F_0^{(2)} = f_0 + f_1 W_2^0 = \sum_{j=0}^{1} f_j (W_2^0)^j
\]

\[
F_1^{(2)} = f_0 + f_1 W_2^1 = \sum_{j=0}^{1} f_j (W_2^1)^j
\]

Let us use \(N = 8\) to illustrate the algorithm for simplicity.
First, break $F_k^{(8)}[W^k_8 | f_0, f_1, \ldots, f_7]$ into even and odd powers as follows (for $0 \leq k \leq 7$)

$$F_k^{(8)}[W^k_8 | f_0, f_1, f_2, \ldots, f_7] = \sum_{j=0}^{3} f_{2j}(W^k_8)^j + W^k_8 \sum_{j=0}^{3} f_{2j+1}(W^k_8)^j$$

---------- (4.10)

Note that

$W^{2k}_8 = \exp(2\pi i(2k)/8) = \exp(2\pi ik/4) = W^k_4$

$W^{k+4}_8 = \exp(2\pi i(k+4)/8) = -\exp(2\pi ik/8) = -W^k_8$

Thus, (4.10) can be evaluated in terms of $F_k^{(4)}$'s and $W^k_4$'s as follows

$$F_k^{(8)}[W^k_8 | f_0, f_1, f_2, \ldots, f_7] = F_k^{(4)}[W^k_4 | f_0, f_2, f_4, f_6] + W^k_8 F_k^{(4)}[W^k_4 | f_1, f_3, f_5, f_7]$$

and

$$F_{k+4}^{(8)}[W^{k+4}_8 | f_0, f_1, f_2, \ldots, f_7] = F_k^{(4)}[W^k_4 | f_0, f_2, f_4, f_6] - W^k_8 F_k^{(4)}[W^k_4 | f_1, f_3, f_5, f_7]$$

for $0 \leq k \leq 3$.

This process can be continued by the same reason. So,

$$F_k^{(4)}[W^k_4 | f_0, f_2, f_4, f_6] = F_k^{(2)}[W^k_2 | f_0, f_4] + W^k_4 F_k^{(2)}[W^k_2 | f_0, f_3]$$

and

$$F_{k+2}^{(4)}[W^{k+2}_4 | f_0, f_2, f_4, f_6] = F_k^{(2)}[W^k_2 | f_0, f_4] - W^k_4 F_k^{(2)}[W^k_2 | f_1, f_3]$$

for $0 \leq k \leq 1$
Direct evaluation is used at the level $N = 2$, i.e.,

$$F_k^{(2)}[W^0_2 | f_0, f_4] = f_0 + W^0_2 f_4 = f_0 + f_4$$

and

$$F_k^{(2)}[W^1_2 | f_0, f_4] = f_0 + W^1_2 f_4 = f_1 - f_4$$

Note: the computer code is presented in Appendix II.

Conclusion:

Now, there are $2^{10}$ sample data per channel. So, it is necessary to calculate $F_0, F_1, F_2, \ldots, F_{1023}$. The above procedure can be used to obtain all $F_k$'s. Then, by using (4.15), the discrete Fourier function can be obtained to approximate the sample data.

4.3 Discussion

(1) Because the cut off frequency is 30 Hz (low pass), it is unnecessary to calculate all the discrete Fourier coefficients. Only those Fourier coefficients which correspond to frequencies less than 30 Hz are needed.

(2) The main reason for choosing 30 Hz to be the cut off frequency is that the filtered data will represent the longest steady state. Fig. 4.3 ~ Fig. 4.5 are shown to compare the filtered data obtained by using different cut off
Fig. 4.3 Cut off frequency 20 Hz

Fig. 4.4 Cut off frequency 30 Hz
Fig. 4.5 Cut off frequency 60 Hz

Fig. 4.6 Frequency distribution for model A
frequencies for the same raw balance data.

(3) The flow in the tunnel is initiated by suddenly opening the sliding-sleeve valve. This action causes the tunnel to vibrate. Because the balance is mounted to an access port in the test section, the tunnel vibration is transmitted to the balance. The balance is a sensitive force and moment measurement instrument. We need to analyse the frequency distribution of the raw balance data to determine the influence of the tunnel vibration.

The computer code presented in Appendix III was used to transform the raw balance data from time domain to frequency domain. The computer code is based on the FFT theorem.

From Fig 4.6. ~ Fig 4.8. (frequency domain), we can see that there are two significant vibration modes for channel 1. The vibration modes for other channels can be obtained by the same procedure. They are presented in Table 3.

Table 3 Vibration Modes (all dimensions are in Hz)

<table>
<thead>
<tr>
<th></th>
<th>Channel 1</th>
<th>Channel 3</th>
<th>Channel 5</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Model A</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1st mode</td>
<td>230</td>
<td>300</td>
<td>260</td>
</tr>
<tr>
<td>2nd mode</td>
<td>650</td>
<td>800</td>
<td>380</td>
</tr>
<tr>
<td><strong>Model B</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1st mode</td>
<td>230</td>
<td>350</td>
<td>250</td>
</tr>
<tr>
<td>2nd mode</td>
<td>770</td>
<td>950</td>
<td>750</td>
</tr>
<tr>
<td><strong>Zero loading</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1st mode</td>
<td>320</td>
<td>300</td>
<td>300</td>
</tr>
<tr>
<td>2nd mode</td>
<td>650</td>
<td>760</td>
<td>380</td>
</tr>
</tbody>
</table>
Fig. 4.7 Frequency distribution for model B

Fig. 4.8 Frequency distribution for zero loading
Note:

(1) The definition of zero loading is
   (a) Model A is mounted in the tunnel.
   (b) The sliding sleeve valve is opened with atmospheric pressure air in the tunnel (i.e. there is no aerodynamic force acting on the balance. Only the tunnel vibration influences the output of the balance).

(2) The frequency distribution of channel 2 and channel 4 are almost the same as channel 1 and channel 3, respectively.

From Table 3, the following conclusions can be obtained.

(i) The frequencies for the first mode represent the influence of the tunnel vibration. The reason is that the frequencies of the first mode for model A, model B and zero loading are almost the same. Thus, the frequencies of the first mode are independent of model change. They only depend on the tunnel vibration. So, the tunnel vibration is the major reason for the oscillation observed in the raw balance data.

(ii) The frequencies for the second mode present the influence of the model change. The reason is that the frequencies of the second mode for model A and zero loading are almost the same. But, they are different from that for model B. So, we conclude that the tunnel vibration does not
have an influence on the second mode. Instead, use of different models influences the second mode.

Remark:

The oscillation of the balance can be reduced by increasing the stiffness of the balance. But, the increment of the stiffness of the balance can reduce the output of the balance signal. It can make the measurement of the aerodynamic forces and moments more difficult. So, there is a tradeoff between the balance oscillation and the accuracy of the output of the balance signal.
CHAPTER V

DATA REDUCTION and DATA REPRESENTATION

This chapter describes how the test data are reduced through the computer codes and how they are presented and analyzed. Data from the two vortex generators described in Section 2.3 are used to illustrate the procedure.

5.1 Data Reduction Procedure

The raw data obtained during the run is reduced to coefficient form by the following procedure.

(1) Convert the raw data from PSP file to ASCII file.

(2) Separate ASCII file into balance data and pressure data.

(3) From the reduction of the pressure data, the time, static pressure, total pressure and Mach number are obtained.

(4) Filter raw balance data by using the filter code described in Section 4.2. Please see Appendix I for computer code.

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(5) By using the iterated equation (3.1) which will be discussed in Appendix III, NF, MZ, CF, MZ, PM are obtained.

(6) Because there is an adaptor between the model and the balance (see Fig. 2.8), we need to transform the MX and MZ from the origin at the surface of the balance to the root of the model. The NF, CF, and PM will not be affected by the presence of the adaptor. The conversion equations are described as follow,

\[ MX' = MX - TH \times NF \]  \hspace{2cm} (5.1)

\[ MZ' = MZ - TH \times CF \]  \hspace{2cm} (5.2)

Note that there is a baseplate for Model A and a base for Model B. So, TH is equal to the sum of the thickness of the baseplate and the thickness of the adaptor.

<table>
<thead>
<tr>
<th></th>
<th>Model A</th>
<th>Model B</th>
</tr>
</thead>
<tbody>
<tr>
<td>TH (in)</td>
<td>1</td>
<td>0.71875</td>
</tr>
</tbody>
</table>

(7) Convert the forces and moments from body axis to stability axis. The following equations do the conversion. (Note that we use the left-hand rule for the coordinate transformation).
L (Lift) = NF * COSα - CF * SINα

D (Drag) = NF * SINα + CF * COSα

\[ 1 \text{ (Rolling Moment)} = MX' * \text{COSα} - MZ' * \text{SINα} \]

n (Yawing Moment) = MX' * SINα + MZ' * COSα

where α is the angle of attack.

(8) Reduce the forces and moments to coefficient form.

\[ L = \frac{1}{2} C_L \gamma P M^2 S \]

\[ D = \frac{1}{2} C_D \gamma P M^2 S \]

\[ \dot{1} = \frac{1}{2} C_1 \gamma P M^2 S b \]

\[ m \text{ (Pitching Moment)} = \frac{1}{2} C_m \gamma P M^2 S c \]

\[ n = \frac{1}{2} C_n \gamma P M^2 S b \]

where \( \gamma \) is the specific heat ratio (\( \gamma = 1.4 \)), \( P \) is the static pressure, \( M \) is the Mach number, \( S \) is the model surface area, \( b \) is the model span, and \( c \) is the model chord.

5.2 Data Representation and Discussion

5.2.1 Reynolds Number Effect

Reynolds number can affect \( C_{L_{\text{max}}} \), but has no influence on the variation of lift coefficient with angle of attack in the linear range. This phenomenon is predicted for subsonic
flow.

From Fig. 5.1, we see that the lift coefficients for Re=4\times10^6 are only slightly higher than those for Re=2.5\times10^6. Thus, the Reynolds number appears to have little influence on the variation of lift coefficient with angle of attack within the linear range for the transonic Mach numbers. More experimental data should be obtained for different Reynolds numbers and Mach numbers before making a definite conclusion.

From Fig 5.2, we see that the drag coefficients for Re=4\times10^6 are greater than those for Re=2.5\times10^6 at the same angle of attack. The reason is that the turbulence for Re=4\times10^6 is more severe than that for Re=2.5\times10^6. So, the friction drag for Re=4\times10^6 is greater than that for Re=2.5\times10^6.

5.2.2 Aspect Ratio Effect

From Fig 5.1, we see that the $C_{L\alpha}(\partial C_L/\partial \alpha)$ for a NACA 0012 airfoil is about 0.075. This value is lower than the theoretical value of 0.106, which is predicted for subsonic flow. The major reason is outlined below.

The theoretical value of 0.106 is predicted for a two dimensional airfoil (infinite wing models). But, the data
presented in Fig 5.1 are obtained for finite wing models. The aspect ratio should be considered.

The effect of aspect ratio is to decrease the slope of the lift curve $C_{L\alpha}$ as the aspect ratio decreases (Ref. 8). Usually, the slope of the wing's lift curve ($C_{L\alpha}$) is approximately 0.79 of the slope for the airfoil in the subsonic regime.

5.2.3 Mach Number Effect

From Fig 5.3 and Fig. 5.4, a conclusion can be obtained. The lift coefficients in the transonic regime increase before the force break for a NACA 0006-34 airfoil. But, the lift coefficients decrease before the force break for a NACA 0012-34 airfoil. From this comparison, we know that the lift coefficient distribution can be totally different in the transonic regime when the section thickness ratio increases. Because of the uncertainty in the transonic regime, available experimental data should be used to analyse the performance of the model. This is a major reason for the development of the balance used for this project.

From Fig. 5.4, we see that the lift coefficient decreases when the Mach number increases. This is a reason that the $C_{L\alpha}$ obtained in Fig. 5.1 is lower than the
theoretical value.

From Fig. 5.5 and Fig. 5.6, we see that the lift coefficient decreases when the Mach number increases, especially for higher angle of attack, for both models.

From Fig 5.7, we see that the drag coefficient increases when the Mach number increases. The reason is that the shock-boundary layer interaction will be more severe when the Mach number increases. So, the profile drag increases.

5.2.4 Drag Effect

The drag polar equation is

\[ C_D = C_{D0} + K * C_L^2 \]  \hspace{1cm} \text{(5.5)}

From Fig. 5.8, we can find that the \( C_{D0} \) and \( K \) for model A are smaller than those for model B. The reasons are
(a) There are surface slope discontinuities for Model B. But, there is no surface slope discontinuity for model A. So, the skin friction of model A is much lower than that of model B.
(b) The tip of model A is rounded. But, the tip of model B is sharp. The vortex at the tip of model B is more severe than the vortex at the tip of model A. So, the induced drag for model B is greater than that of model A.
5.2.5 Pitching Moment

Table 5 presents the lift coefficients and pitching moment coefficients for Model A and Model B at $M=0.78$ and $Re=4\times10^6$.

From this Table, it can be concluded that Model A has smaller pitching moment coefficients than Model B, but has greater lift coefficients than Model B at the same angle of attack.

The pitching moment coefficients shown in Table 5 are measured with respect to the center of the chord. Because they are all positive, that means the center of pressure is located in the first half of the chord (i.e. $0 \leq X_{c.p.} \leq \frac{C}{2}$). We can transform the pitching moment reference point from the
center of the chord to the leading edge. The transformed data are shown in Table 6.

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>0°</th>
<th>4°</th>
<th>8°</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_{m1}$</td>
<td>0.001599</td>
<td>0.0142</td>
<td>0.0116</td>
</tr>
<tr>
<td>$C_{m2}$</td>
<td>-0.01465</td>
<td>-0.12949</td>
<td>-0.2529</td>
</tr>
</tbody>
</table>

Note:

$C_{m1}$: pitching moment coefficient with respect to the center of the chord.

$C_{m2}$: pitching moment coefficient with respect to the leading edge.

From Table 6, we see that the absolute value of the pitching moment coefficients with respect to the leading edge are greater than those with respect to the center of the chord at the same angle of attack. A interesting conclusion can be obtained from this phenomenon.

The center of pressure (c.p.) and the aerodynamic center (a.c.) are the same for the symmetrical airfoil (i.e. $X_{c.p.} = X_{a.c.}$) (Ref. 10). The aerodynamic center is at $\frac{1}{4}$ chord ($\frac{1}{4}c$) position for a thin airfoil in the subsonic regime. That means the resultant lift acts on the $\frac{1}{4}c$ position. So, the absolute value of $C_{m1}$ should be the same as $C_{m2}$, because the moment arm for $C_{m1}$ is the same as that for $C_{m2}$ ($\frac{1}{4}c$).
From the data shown in Table 6, we see that the absolute value of $C_{m2}$ is greater than the absolute value of $C_{m1}$. That means the moment arm for $C_{m2}$ is greater than that for $C_{m1}$. So, $X_{a.c.}$ is between $\frac{1}{4}c$ and $\frac{1}{2}c$. That means the aerodynamic center moves backward at the transonic regime. This phenomenon is predicted in some Flight Mechanics books (References 8 and 10).

5.2.6 Yawing and Rolling Moment Coefficients

The yawing and rolling moment coefficients are important for an aircraft, but are not significant for the test of the rotor blade configurations used in this test. So, these data are not presented in this thesis.
**Figure 5.1** Variation of lift coefficient with angle of attack

**Figure 5.2** Variation of drag coefficient with angle of attack
Fig. 5.3 Variation of lift coefficient with Mach number for NACA 0006-34 airfoil (Ref. 9)

Fig. 5.4 Variation of lift coefficient with Mach number for NACA 0012-34 airfoil (Ref. 9)
Fig. 5.5 Variation of lift coefficient with angle of attack for three different Mach numbers, Model A

Fig. 5.6 Variation of lift coefficient with angle of attack for three different Mach numbers, Model B
Fig. 5.7 Variation of drag coefficient with Mach number for two different Reynolds number

Fig. 5.8 Variation of lift coefficient with drag coefficient
CHAPTER VI

CONCLUSIONS and RECOMMENDATIONS

A description of the balance was presented in Chapter II. The balance functioned well for this project. The balance and data acquisition system combination provide a fast, convenient and accurate (Appendix IV) method of determining model force and moments.

The calibration procedure is complex, but easy to follow. The sensitivity constants and interaction coefficients should be periodically recalibrated.

The FFT theorem is a very powerful tool. Recently, a lot of work has been done on this theorem, especially for determining the structural frequency response.

The accuracy of balance data can be improved by use of an analog filter before entering the data acquisition system. This would allow use of higher amplifier gains to obtain more accurate balance data, without saturating the amplifier.

In Chapter V, the data reduction and data analysis were explained. All equipment performed satisfactorily under operating conditions and provided useable data for analysis.
More investigation should be done to investigate the Reynolds number effect and the difference between the lift coefficients for both models.
Appendix I

COMPUTER ALGORITHM FOR DATA REDUCTION
This appendix is adopted from reference 4. Please refer to it for further information.

The sensitivity constants for each component of the balance are

NF: 630.51702 (lb/mv)
MX: 147.95088 (lb-in/mv)
CF: 11.063526 (lb/mv)
MZ: 137.74105 (lb-in/mv)
PM: 24.8847 (lb-in/mv)

The uncorrected forces and moments (F^{(0)}) are determined by multiplying reduced balance data by their sensitivity constants, respectively. For example

The reduced balance data for 500 lb at loading point A on +NF plane are

NF: 0.797 (mv)
MX: 6.223 (mv)
CF: 0.203 (mv)
MZ: -0.021 (mv)
PM: -0.037 (mv)

The uncorrected $F^{(0)}$ is

\[ F^{(0)}_0 (NF) = 0.797 \times 630.51702 = 502.52 \text{ (lb)} \]
\[ F^{(0)}_1 (MX) = 6.223 \times 147.95088 = 920.70 \text{ (lb-in)} \]
\[ F^{(0)}_2 (CF) = 0.203 \times 11.063526 = 2.246 \text{ (lb)} \]
\[ F^{(0)}_3 (MZ) = -0.021 \times 137.74105 = -2.89 \text{ (lb-in)} \]
\[ F^{(0)}_4 (PM) = -0.037 \times 24.8847 = -0.92 \text{ (lb-in)} \]

By substituting $F^{(0)}$ into (3.1), the corrected forces and moments can be obtained by a number of iterations. Usually, the number of iteration is 9 - 13.

The balance matrix $M$ in (3.1) is shown below.

<table>
<thead>
<tr>
<th></th>
<th>NF</th>
<th>MX</th>
<th>CF</th>
<th>MZ</th>
<th>PM</th>
</tr>
</thead>
<tbody>
<tr>
<td>NF</td>
<td>1</td>
<td>1.85377</td>
<td>3.81\times10^{-3}</td>
<td>-5.9\times10^{-3}</td>
<td>-3.6\times10^{-3}</td>
</tr>
<tr>
<td>NF^2</td>
<td>7.97\times10^{-6}</td>
<td>-3.5\times10^{-6}</td>
<td>1.82\times10^{-6}</td>
<td>-1.7\times10^{-6}</td>
<td>9.75\times10^{-6}</td>
</tr>
<tr>
<td>MX</td>
<td>-0.0978</td>
<td>1</td>
<td>2.16\times10^{-3}</td>
<td>-4.1\times10^{-3}</td>
<td>-3.1\times10^{-3}</td>
</tr>
<tr>
<td>MX^2</td>
<td>1.58\times10^{-5}</td>
<td>-1.5\times10^{-5}</td>
<td>0</td>
<td>-1.7\times10^{-6}</td>
<td>3.11\times10^{-7}</td>
</tr>
<tr>
<td>NF \times MX</td>
<td>-3.2\times10^{-5}</td>
<td>2.23\times10^{-5}</td>
<td>-2.8\times10^{-7}</td>
<td>5.19\times10^{-6}</td>
<td>-3.4\times10^{-6}</td>
</tr>
<tr>
<td>PM</td>
<td>-0.037</td>
<td>8.72\times10^{-3}</td>
<td>3.62\times10^{-3}</td>
<td>-0.022</td>
<td>1</td>
</tr>
<tr>
<td>PM^2</td>
<td>-2.4\times10^{-5}</td>
<td>2.08\times10^{-5}</td>
<td>2.82\times10^{-7}</td>
<td>1.76\times10^{-6}</td>
<td>-1.3\times10^{-6}</td>
</tr>
<tr>
<td>NF \times PM</td>
<td>1.42\times10^{-4}</td>
<td>-1.7\times10^{-4}</td>
<td>5.21\times10^{-5}</td>
<td>9.27\times10^{-5}</td>
<td>-7.8\times10^{-5}</td>
</tr>
<tr>
<td>CF</td>
<td>-0.1998</td>
<td>-0.101</td>
<td>1</td>
<td>1.909</td>
<td>7.89\times10^{-3}</td>
</tr>
<tr>
<td>CF^2</td>
<td>3.51\times10^{-4}</td>
<td>2.47\times10^{-4}</td>
<td>1.85\times10^{-5}</td>
<td>-7.7\times10^{-5}</td>
<td>1.38\times10^{-5}</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>-9.9*10</td>
<td>2.22*10</td>
<td>1</td>
<td>-8.3*10</td>
</tr>
<tr>
<td>-------</td>
<td>-----</td>
<td>---------</td>
<td>---------</td>
<td>------</td>
<td>---------</td>
</tr>
<tr>
<td>MZ$^2$</td>
<td>-3.5*10^{-4}</td>
<td>-8.3*10^{-5}</td>
<td>0</td>
<td>-3.9*10^{-5}</td>
<td>-2.1*10^{-5}</td>
</tr>
<tr>
<td>MZ * MZ</td>
<td>3.52*10^{-4}</td>
<td>8.26*10^{-5}</td>
<td>1.2*10^{-5}</td>
<td>1.15*10^{-4}</td>
<td>9.02*10^{-5}</td>
</tr>
</tbody>
</table>
Appendix II

COMPUTER CODE FOR FILTER
C******************************************************************************
C* This code uses FFT theorem to interpolate function f
C* B(I) : midpoint.
C* C1(I),C2(I),C3(I),C4(I),C5(I) : data for each channel
C* C(I),CC(I) : complex variables.
C* IW : cut off frequency (Hz).
C* N : number of data.
C* The maximum number of data for each channel is 1024
C* The maximum cut off frequency is 1000 Hz
C******************************************************************************

DIMENSION B(1025)
COMMON C(11,1025),CC(11,1025),C1(1025),C2(1025),
+C3(1025),C4(1025)
COMMON C5(1025)
COMPLEX C,CC,X
OPEN(1,FILE='BALYOU.DAT')
N=1024
MT=10
WRITE (*,*) 'INPUT THE CUT OFF FREQUENCY (Hz)'
READ (*,41)IW
41 FORMAT(I3)
IW=INT(IW*0.512)
IF (IW.GT.1000) THEN
WRITE (*,*) ' THE MAXIMUM CUT OFF FREQUENCY IS
+1000 (Hz) '
ENDIF
DO 20 I=0,127
DO 15 J=1,8
KC=8*I+J
READ(1,5) C1(KC),C2(KC),C3(KC),C4(KC),C5(KC)
5 FORMAT(2X,5(F6.1,2X))
15 CONTINUE
20 CONTINUE
10 CONTINUE
DO 25 I=1,1024
B(I)=(0.5*I)/1000
25 CONTINUE
DO 16 NN=1,5
DO 17 I=1,1024
IF (NN.EQ.1) THEN
C(I,I)=CMPLX(C1(I),0.0)
ELSE IF (NN.EQ.2) THEN
C(I,I)=CMPLX(C2(I),0.0)
ELSE IF (NN.EQ.3) THEN
C(I,I)=CMPLX(C3(I),0.0)
ELSE IF (NN.EQ.4) THEN
C(I,I)=CMPLX(C4(I),0.0)
ELSE IF (NN.EQ.5) THEN
C(I,I)=CMPLX(C5(I),0.0)
ENDIF
ENDIF
17 CONTINUE
CALL FFT(N,MT)
CALL SOLVE(B,N,MT,IW,NN)
WRITE (*,18)NN
18 FORMAT(1X,I2)
16 CONTINUE
CLOSE (1)
STOP
END

C******************************************************************************
C*                        *   This subroutine can get Fourier coefficients for  *
C*    each channel.        *   *                                           *
C*                        *                                           *
C******************************************************************************

SUBROUTINE FFT(N,MT)
COMMON C(11,1025),CC(11,1025)
COMPLEX C,CC,X
P=3.1415926
DO 100 M=0,(MT-1)
   M1=(2**M)-1
   DO 90 J=0,M1
      X=CMPLX(0.0,2*P*J/(2**(M+1)))
      N1=J*N/(2**(M+1))
      N2=J*N/(2**M)
      N3=N/(2**(M+1))
      N4=N/2
      M2=N/(2**(M+1))-1
      DO 80 I=0,M2
         C((M+2),(I+N1+1))=C((M+1),(I+N2+1))+CEXP(X)*C(M+1,
                          +(I+1+N2+N3))
         C((M+2),(I+N1+N4+1))=C((M+1),(I+N2+1))-CEXP(X)*
                          +C((M+1),(I+1+N2+N3))
     80 CONTINUE
90 CONTINUE
100 CONTINUE
RETURN
END

C******************************************************************************
C*                        *   This subroutine can set up approximated function f  *
C*                        *                                           *
C******************************************************************************

SUBROUTINE SOLVE(B,N,MT,IW,NN)
DIMENSION B(1025)
COMMON C(11,1025),CC(11,1025),C1(1025),C2(1025),
     +C3(1025),C4(1025)
COMMON C5(1025)
COMPLEX C,CC,H,X
OPEN (3,FILE='EVER.DAT',STATUS='NEW')
P=3.1415926
DO 110 I=2,N
CC(MT+1,I)=CONJG(C((MT+1),I))
110 CONTINUE
IF ((N/2).GT.IW) THEN
   III=IW
ELSE
   III=N/2
ENDIF
DO 130 I=0,(N-1)
H=CMPLX(0.0,0.0)
DO 120 K=-(III+1),(III-1)
   X=CMPLX(0.0,-2*P*K*B(I+1)/0.512)
   IF (K.LT.0) THEN
      H=H+CC((MT+1),(-K+1))*CEXP(X)/N
   ENDIF
   IF (K.GE.0) THEN
      H=H+CC((MT+1),(K+1))*CEXP(X)/N
   ENDIF
120 CONTINUE
Y=REAL(H)
IF (NN.EQ.1) THEN
   C1(I+1)=INT(Y*100)/100.0
ELSE IF (NN.EQ.2) THEN
   C2(I+1)=INT(Y*100)/100.0
ELSE IF (NN.EQ.3) THEN
   C3(I+1)=INT(Y*100)/100.0
ELSE IF (NN.EQ.4) THEN
   C4(I+1)=INT(Y*100)/100.0
ELSE IF (NN.EQ.5) THEN
   C5(I+1)=INT(Y*100)/100.0
ENDIF
130 CONTINUE
IF (NN.EQ.5) THEN
   DO 140 I=1,1024
      T=I*0.5
      WRITE (*,145)T,C1(I),C2(I),C3(I),C4(I),C5(I)
145 FORMAT(1X,6(F7.2,3X))
140 CONTINUE
ENDIF
CLOSE (3)
RETURN
END
APPENDIX III

COMPUTER CODE FOR TRANSFORMING DATA FROM TIME DOMAIN TO
FREQUENCY DOMAIN
This code uses FFT theorem to transform data from time domain to frequency.

B(I): midpoint.
CN(5,1025): data for each channel.
C(I),CC(I): complex variable.
HZ: frequency.
N: number of data.
The maximum number of data for each channel is 1024.

DIMENSION B(1025)
COMMON C(11,1025),CC(11,1025),CN(5,1025)
COMPLEX C,CC,X
OPEN(1,FILE='BALYOU.DAT')
OPEN(2,FILE='MAG.DAT',STATUS='NEW')
N=1024
MT=10
DO 20 I=0,127
  DO 15 J=1,8
    KC=8*I+J
    READ(1,5) CN(1,KC),CN(2,KC),CN(3,KC),CN(4,KC),CN(5,KC)
  5 FORMAT(2X,5(F6.1,2X))
  15 CONTINUE
20 CONTINUE
DO 25 I=1,1024
  B(I)=(0.5*I)/1000
25 CONTINUE
DO 16 NN=1,5
  DO 31 I=1,1024
    C(I,I)=CMPLX(CN(NN,I),0.0)
  31 CONTINUE
CALL FFT(NN,N,MT)
WRITE (*,18)NN
18 FORMAT(1X,I2)
16 CONTINUE
DO 70 I=1,512
  HZ=I/0.512
  WRITE (2,75)HZ,CN(1,I),CN(2,I),CN(3,I),CN(4,I),CN(5,I)
75 FORMAT(1X,F7.1,2X,5(F10.1,2X))
70 CONTINUE
CLOSE (1)
CLOSE (2)
STOP
END

This subroutine calculates the Fourier coefficients corresponding to each HZ, respectively.
SUBROUTINE FFT(NN,N,MT)
COMMON C(11,1025),CC(11,1025),CN(5,1025)
COMPLEX C,CC,X
P=3.1415926
DO 100 M=0,MT-1
M1=(2**M)-1
DO 90 J=0,M1
X=CMPLX(0.0,2*P*J/(2**(M+1)))
N1=J*N/(2**(M+1))
N2=J*N/(2**M)
N3=N/(2**(M+1))
N4=N/2
M2=N/(2**(M+1))-1
DO 80 I=0,M2
C((M+2),(I+N1+1))=C((M+1),(I+N2+1)) + CEXP(X)*C(M+1,
+ (I+1+N2+N3))
C((M+2),(I+N1+N4+1))=C((M+1),(I+N2+1)) - CEXP(X)*
+C((M+1),(I+1+N2+N3))
80 CONTINUE
90 CONTINUE
100 CONTINUE
DO 110 I=1,512
TLL=REAL(C((MT+1),I))
CN(NN,I)=INT(TLL)
110 CONTINUE
RETURN
END
Appendix IV
ERROR ANALYSIS
By adopting the calibration data in reference 4, the error analysis can be performed. The following data are adopted from the Accuracy Check section.

<table>
<thead>
<tr>
<th></th>
<th>Appl. Load</th>
<th>500</th>
<th>500</th>
<th>-500</th>
<th>-500</th>
<th>0.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>NF</td>
<td>Corr. Load</td>
<td>499.99</td>
<td>500.76</td>
<td>-500.40</td>
<td>-500.24</td>
<td>-0.0</td>
</tr>
<tr>
<td></td>
<td>Error</td>
<td>-0.01</td>
<td>0.76</td>
<td>-0.40</td>
<td>-0.24</td>
<td>0.0</td>
</tr>
<tr>
<td>MX</td>
<td>Appl. Load</td>
<td>0.0</td>
<td>1000</td>
<td>-500</td>
<td>-1000</td>
<td>0.0</td>
</tr>
<tr>
<td></td>
<td>Corr. Load</td>
<td>0.01</td>
<td>1001.27</td>
<td>-502.10</td>
<td>-1002.02</td>
<td>0.01</td>
</tr>
<tr>
<td></td>
<td>Error</td>
<td>0.01</td>
<td>1.27</td>
<td>-2.10</td>
<td>-2.02</td>
<td>0.01</td>
</tr>
<tr>
<td>PM</td>
<td>Appl. Load</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td></td>
<td>Corr. Load</td>
<td>0.0</td>
<td>0.43</td>
<td>-0.40</td>
<td>-0.40</td>
<td>-0.0</td>
</tr>
<tr>
<td></td>
<td>Error</td>
<td>0.0</td>
<td>0.43</td>
<td>-0.40</td>
<td>-0.40</td>
<td>0.0</td>
</tr>
<tr>
<td>MZ</td>
<td>Appl. Load</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td></td>
<td>Corr. Load</td>
<td>0.02</td>
<td>-0.04</td>
<td>0.10</td>
<td>0.09</td>
<td>-0.0</td>
</tr>
<tr>
<td></td>
<td>Error</td>
<td>0.02</td>
<td>-0.04</td>
<td>0.10</td>
<td>0.09</td>
<td>0.0</td>
</tr>
<tr>
<td>CF</td>
<td>Appl. Load</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>75</td>
</tr>
<tr>
<td></td>
<td>Corr. Load</td>
<td>-0.01</td>
<td>0.03</td>
<td>-0.06</td>
<td>-0.06</td>
<td>75</td>
</tr>
<tr>
<td></td>
<td>Error</td>
<td>-0.01</td>
<td>0.03</td>
<td>-0.06</td>
<td>-0.06</td>
<td>0.0</td>
</tr>
<tr>
<td></td>
<td>Appl. Load</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>-----</td>
<td>------------</td>
<td>-----</td>
<td>-----</td>
<td>-----</td>
<td>-----</td>
<td>-----</td>
</tr>
<tr>
<td>NF</td>
<td>Corr. Load</td>
<td>0.25</td>
<td>-0.26</td>
<td>-0.25</td>
<td>-0.04</td>
<td>0.56</td>
</tr>
<tr>
<td></td>
<td>Error</td>
<td>0.25</td>
<td>-0.26</td>
<td>-0.25</td>
<td>-0.04</td>
<td>0.56</td>
</tr>
<tr>
<td>MX</td>
<td>Appl. Load</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td></td>
<td>Corr. Load</td>
<td>-0.63</td>
<td>0.61</td>
<td>0.66</td>
<td>-0.53</td>
<td>-1.17</td>
</tr>
<tr>
<td></td>
<td>Error</td>
<td>-0.63</td>
<td>0.61</td>
<td>0.66</td>
<td>-0.53</td>
<td>-1.17</td>
</tr>
<tr>
<td>PM</td>
<td>Appl. Load</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td></td>
<td>Corr. Load</td>
<td>-0.13</td>
<td>0.13</td>
<td>0.13</td>
<td>139.97</td>
<td>-140.15</td>
</tr>
<tr>
<td></td>
<td>Error</td>
<td>-0.13</td>
<td>0.13</td>
<td>0.13</td>
<td>-0.03</td>
<td>-0.15</td>
</tr>
<tr>
<td>MZ</td>
<td>Appl. Load</td>
<td>75</td>
<td>-75</td>
<td>-150</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td></td>
<td>Corr. Load</td>
<td>75.17</td>
<td>-75.17</td>
<td>-150.17</td>
<td>-0.10</td>
<td>-0.12</td>
</tr>
<tr>
<td></td>
<td>Error</td>
<td>0.17</td>
<td>-0.17</td>
<td>-0.17</td>
<td>-0.10</td>
<td>-0.12</td>
</tr>
<tr>
<td>CF</td>
<td>Appl. Load</td>
<td>75</td>
<td>-75</td>
<td>-75</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td></td>
<td>Corr. Load</td>
<td>75.02</td>
<td>-75.17</td>
<td>-75.02</td>
<td>0.13</td>
<td>0.13</td>
</tr>
<tr>
<td></td>
<td>Error</td>
<td>0.02</td>
<td>-0.17</td>
<td>-0.02</td>
<td>0.13</td>
<td>0.13</td>
</tr>
</tbody>
</table>

The standard deviation \( \sigma \) for each force and moment can be obtained by using the above data. The \( \sigma \) is defined as

\[
\sigma = \left[ \frac{\sum_{i=1}^{N} (\text{Corr. Load} - \text{Appl. Load})^2}{N - 1} \right]^{0.5}
\]

where \( N \) is number of data.

\[\sigma_{\text{NF}} = 0.1268 \text{ (lb)}\]
\[\sigma_{\text{MX}} = 0.39996 \text{ (lb-in)}\]
\[\sigma_{\text{PM}} = 0.0845 \text{ (lb-in)}\]
\[
\sigma_{MZ} = 0.0402 \text{ (lb-in)} \\
\sigma_{CF} = 0.023 \text{ (lb)}
\]

The \( \sigma \) can be translated to a percentage value of full scale load in the following manner:

\[
\text{NF: } \frac{0.1268 \times 100\%}{500} = 0.02536\% \\
\text{MX: } \frac{0.39996 \times 100\%}{1000} = 0.039996\% \\
\text{PM: } \frac{0.0845 \times 100\%}{140} = 0.060357\% \\
\text{MZ: } \frac{0.0402 \times 100\%}{150} = 0.0268\% \\
\text{CF: } \frac{0.023 \times 100\%}{75} = 0.03066\%
\]

From normal error distribution theory, more than 99% of all data will fall within the range 3\( \times \sigma \) of the applied load as \( N \) approaches infinity (Ref. 11). Because the standard deviations (\( \sigma \)) obtained for this balance are so small, accurate data can be obtained by using this balance and the iterated equation.
REFERENCES


