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ARC HEATER PERFORMANCE, CALIBRATION, AND
DATA CORRELATION

The members of the Committee approve the masters
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ARC HEATER PERFORMANCE, CALIBRATION, AND
DATA CORRELATION

by

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The University of Texas at Arlington in Partial Fulfillment
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ABSTRACT

ARC HEATER PERFORMANCE, CALIBRATION, AND DATA CORRELATION

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A continuous-flow, arc-heated hypersonic propulsion test facility has been developed at The University of Texas at Arlington. Descriptions of the arc heater, nozzle, test section, diffuser, and vacuum tank, together with the supporting electrical, pneumatic, cooling water, instrumentation, and data acquisition/control systems are provided. The energy balance method is described and used to obtain the performance map. Tests show similar trends to other known operating arc heaters. The performance map indicated that enthalpy levels of up to 5,700 kJ/kg at 5.5 ATM could be obtained. Test duration based on 0.12 kg/s injection mass flow rate is approximately 30 seconds. Data correlation, statistical analysis, and uncertainty analysis are performed and discussed.
# TABLE OF CONTENTS

ACKNOWLEDGMENTS ................................................................. III

ABSTRACT ........................................................................ IV

LIST OF FIGURES .............................................................. VII

LIST OF TABLES ................................................................ IX

Chapter

1. INTRODUCTION ....................................................................... 1

1.1. Historical Background .................................................. 1

1.2. Theory of Operation ...................................................... 2

1.3. UT Arlington Arc Heater Facility .................................... 3

2. FACILITY SETUP ................................................................ 4

2.1. Arc Heater and Test Section Assembly ......................... 5

2.2. Electrical System .......................................................... 6

2.3. High-Pressure, Deionized Cooling Water System .......... 6

2.4. Pneumatic System .......................................................... 7

2.5. Instrumentation System ................................................. 8

2.6. Data Acquisition/Control System ................................. 12

2.7. Safeguard Circuit ........................................................... 14
3. ENERGY BALANCE METHOD .......................................... 17

3.1. The Conservation of Energy Equation ............................ 18

4. EXPERIMENTAL PROCEDURE ...................................... 20

4.1. Operational Procedures ........................................... 20

5. RESULTS AND DISCUSSION ......................................... 22

5.1. Data Selection Criterion ........................................... 22

5.2. Data Reduction ..................................................... 23

5.3. Arc Heater Performance Map ...................................... 26

5.4. Numerical Code Comparison ....................................... 34

5.5. Statistical Data Analysis .......................................... 35

5.6. Uncertainty Analysis .............................................. 39

CONCLUSIONS AND RECOMMENDATIONS ............................. 42

Appendix

A. UNCERTAINTY ANALYSIS ........................................... 43

B. EMPIRICAL SCALING RELATIONS ................................. 50

REFERENCES .......................................................... 53
# LIST OF FIGURES

<table>
<thead>
<tr>
<th>Figure</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Segmented Arc Heater</td>
<td>2</td>
</tr>
<tr>
<td>2. Huels Arc Heater</td>
<td>3</td>
</tr>
<tr>
<td>3. Elevation View</td>
<td>4</td>
</tr>
<tr>
<td>4. Schematic Diagram</td>
<td>5</td>
</tr>
<tr>
<td>5. Pneumatic System</td>
<td>8</td>
</tr>
<tr>
<td>6. Instrumentation Locations</td>
<td>9</td>
</tr>
<tr>
<td>7. Data Acquisition/Control System</td>
<td>12</td>
</tr>
<tr>
<td>8. Analog Comparator Circuit</td>
<td>16</td>
</tr>
<tr>
<td>9. Control Volume Schematic</td>
<td>18</td>
</tr>
<tr>
<td>10. Cooling Water Temperature Time Traces</td>
<td>24</td>
</tr>
<tr>
<td>11. Temperature Trace Curve Fit</td>
<td>25</td>
</tr>
<tr>
<td>12. Typical Current, Voltage, and Power Time Traces</td>
<td>26</td>
</tr>
<tr>
<td>13. Current Vs Bulk Enthalpy (AEDC)</td>
<td>27</td>
</tr>
<tr>
<td>14. Power Vs Total Bulk Enthalpy (AEDC)</td>
<td>28</td>
</tr>
<tr>
<td>15. Current Vs Plenum Pressure (AEDC)</td>
<td>29</td>
</tr>
<tr>
<td>16. Arc Current Vs Total Bulk Enthalpy</td>
<td>30</td>
</tr>
<tr>
<td>17. Arc Power Vs Total Enthalpy</td>
<td>30</td>
</tr>
<tr>
<td>18. Arc Current Vs Plenum Pressure</td>
<td>31</td>
</tr>
<tr>
<td>Chapter</td>
<td>Title</td>
</tr>
<tr>
<td>---------</td>
<td>--------------------------------------------</td>
</tr>
<tr>
<td>19</td>
<td>Performance Envelope (constant current and mass flow)</td>
</tr>
<tr>
<td>20</td>
<td>Performance Envelope (constant power and mass flow)</td>
</tr>
<tr>
<td>21</td>
<td>Typical Data Scatter</td>
</tr>
<tr>
<td>22</td>
<td>Trend Removal</td>
</tr>
<tr>
<td>23</td>
<td>Error Distribution</td>
</tr>
<tr>
<td>24</td>
<td>Arc Current Vs Bulk Enthalpy with Error Bars</td>
</tr>
</tbody>
</table>
LIST OF TABLES

Table                                                                 Page
1. HP3852A Accessories                                             13
2. Program Listing                                                  13
3. Summary of the Empirical Coefficients                           33
4. Instrument Accuracies                                            44
5. Experiment Data                                                  45
CHAPTER 1

INTRODUCTION

1.1 Historical Background

Electric arc heater designs date back to the early 1900s where these devices were used for chemical synthesis. By the late 1950s, electric arc heaters started to play a role in aeronautical testing. The need to test thermal protective materials for long-range ballistic missiles and reentry vehicles pushed the operating envelope of these arc heaters to a higher level. Electric arc heaters are now considered the only feasible method to heat gas to high temperatures for a long duration.\(^1\) Now in the 1990s, the interest has shifted toward hypersonic testing for propulsion, material, and structures. High temperature gas flows with duration of several minutes are useful for testing airframe structures and scramjet engines above Mach 8. An arc heater is a wind tunnel where the test gas is heated to the required plenum enthalpies by means of an electric arc. The heated gas is expanded to hypersonic velocities through a nozzle. Despite their long history of impressive service, the arc heater flow characteristics are relatively unknown.\(^2\) There is a need for extensive arc heater flow characterization. Such a flow characterization will provide an accurate account of the arc jet free-stream, a calibration database to help predict the errors in the experimental data caused by the imperfections in the free-stream, and help validate numerical models. Flow uniformity and contamination limitations in arc heater facilities are just a few of the inherent problems.\(^3\)
1.2 Theory of Operation

Arc heaters heat the gas by transferring heat from the sustained arc within the arc chamber to the gas. There are several methods in practice to sustain the arc. The most common method is to use a pair of low resistance electrodes, cathode and anode, separated by multiple or segmented insulators. The cathode and anode are at a fixed distance apart. Gas is injected all along the path of the arc, and is injected tangentially to help stabilize the arc. These types of arc heaters are called “Segmented” arc heaters. The H3 at AEDC in Tennessee is one of the many segmented arc heaters in operation today.

Another more conventional type of arc heater is the “Huels” arc heater. This type of arc heater has one primary electrode, one primary insulator, and one gas injection point. The arc assumes the natural arc length where the termination point is rotated by the swirling motion of the gas. These types of arc heaters are typically easier to operate for aerodynamic applications because of the simplicity of the design. Both types of arc heater are powered by high voltage direct current (DC) power supply. The arc heater at UT Arlington is of the Huels type.

Figure 1. Segmented Arc Heater.
1.3 UT Arlington Arc Heater Facility

The continuous-flow, arc-heated hypersonic test facility at the Aerodynamics Research Center (ARC) at UT Arlington complements the existing test capabilities at the ARC. The facility provides a continuous-flow test environment of sufficient duration to investigate a class of hypersonic flow phenomena that is difficult to study in short-duration facilities. These phenomena include aerodynamic heating, material erosion and ablation, supersonic combustion, hypersonic engine-airframe integration, and stability of hypersonic propulsion systems. The facility was designed to provide the capability of simulating hypersonic flow conditions for aerodynamic testing, or to simulate supersonic combustor exhaust conditions for hypersonic nozzle testing.
CHAPTER 2

FACILITY SETUP

An elevation view of the test facility configured for hypersonic aerodynamic testing is shown in figure 3, and a schematic diagram showing the major components is presented in figure 4. These include the arc heater, nozzle, test section, diffuser and vacuum tank; and the supporting DC power supply, cooling water system, pneumatic system, and vacuum system. Description of major flow-train components and facility support systems are provided in the following sections.

Figure 3. Elevation View.
2.1 Arc Heater and Test Section Assembly

A number of water-cooled nozzle inserts providing a range of Mach numbers from 1.8 to 4.0 was included with the arc heater when shipped from AEDC. The test section is a standard free-jet design. A maximum nozzle exit diameter of 20.3 cm can be used, and the free-jet test section length from the nozzle exit to the diffuser entrance is 61 cm. The diameter of the test section is 76.2 cm. Diagnostic probes or models can be inserted through a 25 cm diameter porthole on the top of the test cabin or a 40.6 by 61 cm model support plate in the bottom of the test cabin. Optical ports of 25.4 cm diameter are located on each side of the test cabin to allow flow-field visualization via holographic interferometry or standard schlieren photography. The diffuser has a 25.4 cm capture diameter, and reduces to a 22.9 cm constant throat diameter diffuser pipe that is 10.67 diameters in length. The diffuser ducts the flow into a large vacuum tank.
2.2 Electrical System

The DC power supply is a Halmar 1.6 MW, current-regulated, plasma torch system, consisting of two Halmar model FRG-I 6-pulse current regulator and trigger circuits, a 12-SCR bridge rectifier, DC load-stabilizing choke, plasma torch interface unit, and associated control and alarm circuitry, interlocks, isolation transformer and interrupter switch. Three-phase AC power at 2,400 V is supplied to the power supply from the ARC main transformer and switchgear panel. The open circuit voltage of the DC power supply is fed to the terminals of the arc heater through a high-frequency igniter that provides the breakdown voltage necessary to initiate the arc. Maximum steady-state operating conditions are 2,000 V/800 A. Current control regulation is ±1 percent of full scale for a 10 percent line/load variation. The operating voltage, current, and power levels are both displayed on panel meters in the facility control room for on-line monitoring, and transmitted to the facility data acquisition/control system. The igniter circuit produces momentary a high voltage ripple (10,000 Volts) which breaks down the gap between the cathode and anode, providing an ionized path for the power supply discharge to flow through. The igniter is also controlled remotely by using a momentary switch.

2.3 High-Pressure, Deionized Cooling Water System

The system consists of a Water Systems. 1,500 lpm / 2,340 kPa pump station, a high-pressure, closed-loop, deionized piping system connected to the test facility; and a low pressure piping system used to reject heat to cooling tower of the ARC compressor plant. The high-pressure deionized cooling water is piped to and from the supply and discharge manifolds. where separate, parallel cooling water lines are connected to the
anode, cathode, plenum chamber, nozzle, test section/diffuser, and DC power supply. The flow rate, temperature rise and pressure drop for each line are monitored on panel meters in the control room and critical measurements are fed to the facility monitoring circuit and into the facility data acquisition/control system. A separate cooling water line is also available for cooling diagnostic probes and models inserted into the flow stream.

2.4 Pneumatic System

A schematic drawing of the pneumatic system is shown in figure 5. The high-pressure nitrogen system consists of six standard 15 MPa nitrogen bottles connected to a Haskel model 29,498 two-stage, air-driven booster pump. The output from the Haskel pump is used to pressurize a 34.5 MPa storage bottle. The arc heater runs in a blow-down mode. A TESCOM model 26-1221 regulator regulates the discharge from the storage bottle, and a Flow-Dyne critical flow nozzle monitors the flow rate. Low pressure 1,200 kPa control air for the operation of the Haskel boost pump, and for remote activation of the control valves during operation of the arc heater is provided by the building’s low-pressure air supply network. Sizing of the high pressure storage bottle allows for an approximate blow-down time of 30 seconds at a constant nominal flow rate of 0.12 kg/s. The storage bottle requires approximately 15 minutes to recharge, depending on the supply pressure of the commercial bottles.
2.5 Instrumentation System

The facility instrumentation system provides measurements of parameters needed for facility set-up and on-line operational monitoring during testing (figure 6). These measurements include the arc heater voltage, current and power, inlet gas flow rate, pressure and temperature, cooling water flow rates, cooling water temperature rise, and arc heater plenum chamber total pressure.
2.5.1 Water Flow Meter Measurements

The two cooling paths, anode and cathode contain Sponsler water flow meters. The flow meter uses a turbine and a magnetic sensor to indicate its rotation rate. There is also a frequency counter that outputs analog voltage linearly with the turbine’s frequency. However, this analog transmitter signal is severely affected by the high voltage interference and subsequently was not used in the experiment. Since the water mass flow rate was constant during the run, the frequency of each turbine flow meter was measured using a counter before each run. Each turbine flow meter was calibrated giving a constant value of turbine frequency per gallon of water. These values are 150.8 and 192.5
cycles/gal for anode and cathode respectively. Therefore, the water flow rates can be calculated by the following equations:

\[
\dot{m}_{\text{anode}} = \frac{x}{150.8} \\
\dot{m}_{\text{cathode}} = \frac{x}{192.5}
\]

Where \( x \) = Frequency Readings

2.5.2 Nitrogen Injection Mass Flow Rate

The Flow Dyne critical flow venturi measures gas flow rate by using a converging-diverging nozzle. Operating on the principle of critical flow, only the inlet pressure and temperature measurements are needed to determine the flow rate. The flow rate varies linearly with the upstream pressure and is not affected by downstream pressure fluctuations. The upstream pressure was regulated by a dome regulator. The flow rate calculation follows the following equation:

\[
\dot{m}_g = K \cdot \frac{P_1}{\sqrt{T_1}}
\]

where \( K \) = the flow coefficient curve for a given inlet pressure and nozzle.

Since the flow coefficient curve (K) from the manufacturer was found to be nearly constant within the range of inlet pressure for this experiment, it was assumed to be a constant.

2.5.3 Gas and Cooling Water Temperature

Temperature measurements are made using Omega Type T Thermal Couple (copper and constantan). Gas temperature measurements used a special thermal couple
with exposed junction for faster response. An ice bath was used as a reference junction so that the readings are absolute. The voltage readings due to the thermoelectric effects can be converted to Kelvin by the following equation supplied by Omega:

\[ T = 273.16 + a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4 + a_5x^5 + a_6x^6 + a_7x^7 \]

Where \( x \) = voltage reading in mV

\[ a_0 = 0.100860910 \quad a_5 = 6.97688e+11 \]
\[ a_1 = 25727.94369 \quad a_6 = -2.66192e+13 \]
\[ a_2 = -767345.8295 \quad a_7 = 3.94078e+14 \]
\[ a_3 = 78025595.81 \]

2.5.4 Pressure Measurements

All pressure measurements were made using Omega pressure transducers. Each transducer has four leads, two for excitation and another two for the analog voltage signal. The full-scale pressure was selected based on the expected values of pressure readings. Full-scale pressure would normally correspond to an output voltage of 100 mV DC.

2.5.5 Power Supply Voltage and Current

The Halmar power supply control board provided proportional 10 Volt analog output for recording the voltage and current. Full-scale outputs correspond to 2,650 Volts and 800 Amps respectively. The power supply uses a shunt to measure the current supplied to the arc heater. The voltage drop across this low resistance shunt is then filtered and amplified by the control board. The voltage across the arc heater is measured
by a voltage divider. This signal is also filtered and amplified. The power signal is believed to be the result of a multiplier circuit built-in on the control board, which receives inputs from both the voltage and current signals.

2.6 Data Acquisition/Control System

The data acquisition system for the test facility consists of a Hewlett-Packard model 3852A Data Acquisition/Control System, interfaced via an IEEE-488 interface bus (HP-IB) to a HP Vectra host computer (figure 7). The 3852A Mainframe has several plug-in accessories installed. However, not all of them were used in this experiment.

Figure 7. Data Acquisition/Control System.
Table 1. HP3852A Accessories

<table>
<thead>
<tr>
<th>Accessory Name</th>
<th>Application</th>
</tr>
</thead>
<tbody>
<tr>
<td>HP 44710A</td>
<td>• Low Speed Switching</td>
</tr>
<tr>
<td></td>
<td>• Low Voltage</td>
</tr>
<tr>
<td></td>
<td>• 20 Channels</td>
</tr>
<tr>
<td>HP 44713A</td>
<td>• High Speed Switching</td>
</tr>
<tr>
<td></td>
<td>• Low Voltage</td>
</tr>
<tr>
<td></td>
<td>• 24 Channels</td>
</tr>
<tr>
<td>HP 44702B</td>
<td>• High Speed Voltmeter</td>
</tr>
<tr>
<td></td>
<td>• 100kHz Sampling Rate</td>
</tr>
<tr>
<td></td>
<td>• 13 bits Resolution</td>
</tr>
<tr>
<td>HP 44728A</td>
<td>• Relay Actuator</td>
</tr>
<tr>
<td></td>
<td>• 8 Channel</td>
</tr>
<tr>
<td>HP 44701A</td>
<td>• Integrating Voltmeter</td>
</tr>
<tr>
<td>HP 44727A</td>
<td>• D/A Converter</td>
</tr>
<tr>
<td></td>
<td>• 4 Outputs</td>
</tr>
</tbody>
</table>

A sample listing of the HP Basic source code used to gather and store data is shown in table 2.

Table 2. Program Listing

REM HP3852A MAINFRAME PROGRAMMING SOURCE CODE
REM Set current directory
10 MASS STORAGE IS "/ARCHEAT:DOS,C"
REM Define output file name
20 ASSIGN @File TO "CALIB.RAW"
REM Reset the Mainframe
30 OUTPUT 709;"RST"
40 CLEAR SCREEN
REM Turn off display for faster performance
50 OUTPUT 709;"DISP OFF"
REM Declare variables
60 OUTPUT 709;"REAL A(31999)"
70 OUTPUT 709;"REAL T1,T2,T3,T"
80 OUTPUT 709;"INTEGER NUM"
90 REAL A(1:32000)
100 REAL T
REM Tell Mainframe to use slot number 6 voltmeter
110 OUTPUT 709;"USE 600"
REM Tell Mainframe to use ribbon cable
120 OUTPUT 709;"SCANNING ON"
REM Define subroutine
130 OUTPUT 709;"SUB TRY"
140 OUTPUT 709;"CONF DCV"
150 OUTPUT 709;"MEAS DCV,500-519, NSCAN 1600, INTO A"
160 OUTPUT 709;"SUBEND"
REM Wait for user
170 INPUT "PRESS ENTER WHEN READY",X
REM Record starting time
180 OUTPUT 709;"TIME INTO T1"
190 OUTPUT 709;"CALL TRY"
REM Record ending time
200 OUTPUT 709;"TIME INTO T2"
REM Calculate time elapsed
210 OUTPUT 709;"T=T2-T1"
220 OUTPUT 709;"VREAD A"
REM Read data from mainframe
230 ENTER 709;A(*)
240 FOR I = 1 TO 32000
REM Format output to file
250 OUTPUT @File USING "K";A(I)
260 NEXT I
REM Read time from mainframe
270 OUTPUT 709;T
280 ENTER 709;T
REM Display elapsed time for user
290 PRINT "TIME FOR COMPLETE SCAN",T
REM Close output file
300 ASSIGN @File TO *
310 END

2.7 Safeguard Circuit

In the event of an interruption of gas or cooling water flow, the arc heater assembly can rapidly melt due to high temperatures. A dedicated analog circuit referred to as a “safeguard circuit,” was designed to continuously monitor each of the above processes. Every module within the system is capable of shutting off the electricity to the arc heater when any of the measured parameters crosses a threshold. The system also
prevents starting the arc heater until all parameters are satisfied. The reaction time, which is primarily limited by the operating time of the relays involved, is estimated to be less than 30 ms. A module consists of several op-amp integrated circuits functioning as buffers, differential amplifier, non-inverting amplifier, and comparators. This design allows the operator to dial in an allowable tolerance for each process. Operators can select a high shut off or conversely a low shut off by flipping DIP switches on the individual module. Light emitting diodes are mounted on the control panel to inform the operator of the status of each module. Transistor logic signals from each channel are connected in series and are used to control a master relay. This master relay, a solid state relay, is in series with power supply’s control circuitry. A schematic of an individual module is show in figure 8.
Figure 8. Analog Comparator Circuit.
CHAPTER 3

ENERGY BALANCE METHOD

Various diagnostic techniques for the measurement of flow characteristics are available. However, the complete arc heater flow characterization is beyond the scope of this work. Instead, this work is dedicated to the measurement of flow enthalpy and, more specifically, the bulk enthalpy. Because the high temperature of the arc heater plume exceeds the melting temperature of almost any material, direct temperature measurements cannot be made to determine the enthalpy of the gas. However, the total enthalpy of the exiting gas can be obtained indirectly. The performance can be evaluated by defining a control volume around the arc heater and applying the conservation of energy. By measuring all energy sources moving across the control surface, it is possible to resolve the bulk enthalpy of the exiting gas. This procedure is usually referred to as the energy balance method. The disadvantage of this method is that it only gives an indication of the bulk enthalpy and does not indicate the flow enthalpy near the central core of the flow where the model is placed.
3.1 The Conservation of Energy Equation

\[
\frac{\delta}{\delta t} \int \left( \rho \left( u + \frac{1}{2} V^2 \right) \right) dVol + \int \rho \left( h + \frac{1}{2} V^2 \right) \vec{V} \cdot \vec{dA} = \frac{dq}{dt} - \frac{dw}{dt}
\]

Where \( dq/dt \) represents the rate of heat addition and \( dw/dt \) the rate at which work is done.

Figure 9 shows the schematic and control volume containing the arc heater. From the control volume, the rate of work is the electrical power added to the flow, IV or Power.

Letting the subscripts \( A, C, g \) denote anode, cathode and gas respectively. The equation can be reduced as follows:

For steady-state flow,

\[
\frac{\delta}{\delta t} \int \rho \left( u + \frac{V^2}{2} \right) dVol = 0
\]
Since velocity of the injected gas is relatively low at the location where the temperature and mass flow is taken, the kinetic energy can be neglected.

\[
\int_{S} \rho \left( \frac{V_{g}^{2}}{2} \right) V \cdot dA = 0
\]

Assuming 1-D flow, the equation can then be expanded to

\[
IV = -(\dot{m} h)_{in} - (\dot{m} h)_{Ain} - (\dot{m} h)_{Cin} + (\dot{m} h)_{Aout} + (\dot{m} h)_{Cout} + \left( \dot{m} \left( h + \frac{V_{g}^{2}}{2} \right) \right)_{gout}
\]

Applying the conservation of mass to the cooling water and gas gives

\[
IV = \dot{m}_{A} (h_{out} - h_{in}) + \dot{m}_{C} (h_{out} - h_{in}) + \dot{m}_{g} (h_{T} - h_{in})
\]

where

\[
h_{T} = h + \frac{V_{g}^{2}}{2}
\]

Solving for \( h_{T} \)

\[
h_{T} = \frac{1}{\dot{m}_{g}} \left[ IV + \dot{m}_{g} h_{g-in} - \dot{m}_{A} (h_{out} - h_{in}) - \dot{m}_{C} (h_{out} - h_{in}) \right]
\]

For the cooling water and injected gas, \( h = C_{p} \cdot T \)

\[
h_{T} = \frac{1}{\dot{m}_{g}} \left[ IV + (\dot{m}_{C_{p}} T)_{g-in} - (\dot{m}_{C_{p}} (T_{out} - T_{in}))_{A} - (\dot{m}_{C_{p}} (T_{out} - T_{in}))_{C} \right]
\]
CHAPTER 4

EXPERIMENTAL PROCEDURE

4.1 Operational Procedures

The dissociated nitrogen has been known to produce nitric oxide. This gas can quickly dilute the oxygen in the room therefore, it is necessary to provide ventilation. Two ceiling ventilation ports are installed in the lab for this purpose. Before charging the nitrogen storage tank, it may be necessary to route the 175 PSI air to the lab. The air is used to control numerous air-actuated valves and the Haskel pump. The storage tank can be pressurized up to 5,000 PSI. Once the tank is pressurized, adjust the dome regulator to inject at the required injection pressure. Begin running the main cooling water pump and cooling water tower. Once the main water pump ramps up to full speed, adjust the gate valves directly upstream of the anode and cathode for the required cooling water mass flow. This cooling flow rate should allow the water temperature to rise for more than 10 °C but should not exceed 50 °C. Energize the power supply by closing the contacters at the power supply switchgear and at the power supply itself. On the control panel, adjust the current setting dial for the desired arc current. At this point, operations can all be controlled from the control room. It is important that the next few steps be carefully timed. Start the data acquisition system and allow for several scans before the next step. Inject nitrogen and immediately energize the anode and cathode. Press and release the igniter button repeatedly until an arc is sustained.
The safeguard circuit will normally shut down the power supply when the nitrogen pressure in the storage tank is low. However, it is recommended that the operator manually disengage the power supply for safe practice. The arc heater lab room should not be disturbed for several minutes due to harmful gases and dangerous electrical discharge from capacitors. Each equipment can now be shut down safely.
CHAPTER 5

RESULTS AND DISCUSSION

5.1 Data Selection Criterion

During the experiment, data were recorded for more than 30 runs. There is only one criterion for accepting data for further analysis. All parameters must be at steady state. Unfortunately, more than half of the runs did not satisfy the criterion and had to be rejected. Here are some explanations for the runs that did not meet the criterion.

• Arc Stability Problems

The combination of gas mass flow rate and arc current setting may not be compatible, in which case, the arc in the chamber tends to behave erratically. Often, the arc is blown out making the runtime too short for steady state conditions to be attained.

• Ignition Problems

The arc fails to ignite within a given amount of time causing the data to fall beyond the test window. Ignition problems are usually due to insufficient voltage to break down the gap. In many cases, the gas mass flow is too high causing the ion path to be convected too strongly. There are new designs being proposed to modify the ignition process. The design concept involves injecting argon gas at ignition. Argon was chosen because it is relatively easy to create an ionized path compared to nitrogen.
• Constricted Mass Flow Problem

Much of the original gas plumbing proved to be unsuitable. The flow cross-section was too small creating a severe pressure drop. This pressure affected the performance of the dome regulator directly upstream of the critical flow venturi. The mass flow was restricted by the narrow plumbing to values less than what the regulator was set to provide. The result was an unsteady or declining mass flow rate when it varies with the pressure of the gas in the storage tank. The unsteady injection mass flow causes other terms in the energy balance equation to also be unsteady and therefore not suitable for performing energy balance calculations. This problem was circumvented temporarily by setting the regulator at a pressure to give the mass flow rate somewhat below the limit imposed by the restrictive plumbing. The plumbing was later modified.

5.2 Data Reduction

A plot of cooling water temperature vs. time trace for a typical run is shown in figure 10. The output from the thermocouple was troublesome due to the large-scale noise on the signals. This problem is primarily due to the output from the type T thermocouples being very small typically less than 6 mV.
Figure 10. Cooling Water Temperature Time Traces.

Accordingly, subsequent processing of these data is necessary to reduce the scatter in the gas total enthalpy results derived from the energy balance calculations. The first step is to identify the test window of data during the run where the temperature is steady. This normally occurs 2-3 seconds after the arc is ignited. Next, any points outside 3.0 standard deviations for a test window are discarded and replaced with the mean temperature for the window. Finally, a least squares linear curve fit is applied to these data (figure 11). Both the anode and cathode thermocouples use the inlet water manifold as the reference junction (see figure 6). Therefore, the resulting measurement is the temperature rise.
Figure 11. Temperature Trace Curve Fit.

The raw data of the current and voltage measurements also show fluctuations (figure 12), but this is not believed to be noise because the power signal, which comes through the same cable, is smooth. The dynamics of the vortex-stabilized arc does create voltage fluctuations for which the power supply has to compensate to maintain a stable current. Accordingly, the average current is constant, although there are instantaneous fluctuations. Linear least square curve fits were applied to the voltage and current data to reduce them for the total enthalpy calculations. The power signal is very smooth and appears to be heavily damped.
5.3 Arc Heater Performance Map

Since the arc chamber pressure affects the heat transfer rates and subsequently the arc heater efficiency, it is important to note here that these results are based on a 1 square inch nozzle throat area. The results from the initial operation of the facility reveal similar trends to the operation of the arc heater at AEDC. Typical arc heater performance plots from the original calibration at AEDC are shown in figures 13 through 15.
Figure 13. Current Vs Bulk Enthalpy (AEDC).
Figure 14. Power Vs Total Bulk Enthalpy (AEDC).
Figure 15. Current Vs Plenum Pressure (AEDC).

Initial arc heater runs at UT Arlington shows similar results. The total enthalpy is observed to increase almost linearly with both increasing current and power (figures 16 and 17). This indicates that arc heater efficiency varies only slightly over the range of operation. Also decreasing the gas mass flow causes the bulk total enthalpy to rise because of there being less gas to absorb the energy. Plenum pressure shows a weak dependence on input current (figure 18). The primary means of manipulating plenum pressure will be using varying throat size nozzles or varying the injection mass flow rates.
Figure 16. Arc Current Vs Total Bulk Enthalpy.

Figure 17. Arc Power Vs Total Enthalpy.
Figure 18. Arc Current Vs Plenum Pressure.

The performance envelopes for the arc heater have been estimated using empirical scaling relations derived from the data collected. The projected chamber pressure and bulk enthalpy envelope with lines of constant current and mass flow rate is shown in figure 19 for the one square inch nozzle throat area. The corresponding plot with lines of constant power and mass flow is shown in figure 20. From figure 15, it is reasonable to assume that the plenum pressure is a linear function of arc current and injection mass flow rate or

\[ P_r = a(I) + b(m_x) + c. \]
A set of coefficients can be solved for such a linear equation where the square of the difference of plenum pressure \( (P_{T_{\text{Exp}}} \) and empirical plenum pressure \( (P_{T_{\text{Emp}}} \) is minimum or

\[
\sum_{n=1}^{N_{\text{Exp}}} (P_{T_{\text{Exp}}} - P_{T_{\text{Emp}}})^2 \text{ is minimum}
\]

The same conclusion can be drawn from figure 16 where the bulk enthalpy also a linear combination of arc current and mass flow rate,

\[
H_r = a(I) + b(m_g) + c.
\]

Solving for another set of coefficients yields a second linear equation. By substituting varying mass flow rates and varying arc current, a map can be drawn. However, it is believed that the bulk enthalpy is more likely a direct function of the electrical power than it is the arc current due to the nature of a current controlled power supply. A current controlled power supply will hold the arc current constant regardless of the resistance across the electrodes, therefore, the arc voltage and power is a direct function of gas mass flow rate. The accuracy of the plots will obviously improve with a more data. A summary of all the coefficients is shown in table 3. Further discussion on this empirical method is available in the appendix B.
Table 3. Summary of the Empirical Coefficients

<table>
<thead>
<tr>
<th>Equation</th>
<th>a</th>
<th>b</th>
<th>c</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_T = a(I) + b(\dot{m}_g) + c$</td>
<td>0.002351</td>
<td>32.14014</td>
<td>-1.21632</td>
</tr>
<tr>
<td>$H_T = a(I) + b(\dot{m}_g) + c$</td>
<td>4.71518</td>
<td>-5818.91663</td>
<td>2799.42544</td>
</tr>
<tr>
<td>$P_T = a(P) + b(\dot{m}_g) + c$</td>
<td>0.00152</td>
<td>22.70456</td>
<td>-0.32064</td>
</tr>
<tr>
<td>$H_T = a(P) + b(\dot{m}_g) + c$</td>
<td>2.80249</td>
<td>-22952.77277</td>
<td>4651.65521</td>
</tr>
</tbody>
</table>

Figure 19. Performance Envelope (constant current and mass flow).
5.4 Numerical Code Comparison

There are a number of computational codes that model arc heat flow characteristics. One that has been used to model the arc heater in this study is ARCFLO II. This code provides the radial distribution of enthalpy at the exit of the constrictor by using a simplified model for the radiative transport phenomena. The code assumes that the constrictor is a constant diameter cylinder where gas is injected axially. It assumes the flow to be in the thermal equilibrium and uses real gas properties in the constricted arc.
region. ARCFLO II is an extension of the Watson and Pegot method. An accurate arc jet flow calculation relies heavily upon the initial conditions used in the calculations. In previous attempts to numerically simulate the conditions of the arc heater flow, the calculations show the same trend as the experimental data. However, there is significant deviation in the quantitative values.\(^7\) Since the gas is injected tangentially into the arc chamber at the anode-cathode interface, the swirling motion of the gas is difficult to model. In addition, the sudden diameter change along the constrictor would also cause irregular flow patterns. The swirling motion restricts a small region of gas that comes in direct contact with the arc.\(^3\) A larger fraction of gas, which does not enter the discharge, is heated relatively slowly by radiation and conduction from the arc column as it flows through the constrictor. The constrictor geometry and the injection method are more than likely the cause of the discrepancies between the numerical and the experimental results. A better understanding of the flow pattern inside the constrictor is necessary for further numerical code improvements.

5.5 Statistical Data Analysis

Run MAR07-03 was chosen from the facility database because of the number of samples collected within the test window, and because of the nominal settings of arc current and injection pressure. The statistical analysis of this run should be a good guide as to what to expect from other runs. The data consist of \(N=550\) data values with an equally spaced sampling interval of \(\Delta t=0.0824\) sec.
The mean value of the sample data \( \{u_n\} \), \( n = 1, 2, \ldots, N \), is given by

\[
\bar{u} = \frac{1}{N} \sum_{n=1}^{N} u_n
\]

It is convenient for later calculations to transform the sample values \( \{u_n\} \) to a new set of values \( \{x_n\} \) that have a zero sample mean by computing

\[
x_n = x(t_0 + n\Delta t) = u_n - \bar{u}, \quad n = 1, 2, \ldots, N
\]

so that all subsequent values will be stated in terms of the transformed data values \( \{x_n\} \), where \( \bar{x} = 0 \).

The standard deviation of the transformed sample data \( \{x_n\} \) is given by\(^8\)

\[
s = \left[ \frac{1}{N-1} \sum_{n=1}^{N} x_n^2 \right]^{\frac{1}{2}}
\]

Note that the quantity \( s \) and \( s^2 \) are estimates of the standard deviation \( \sigma_x \) and the variance \( \sigma_x^2 \), respectively. For convenience, it is desirable to further standardize the data by transforming the values \( \{x_n\} \) to a new set of values \( \{z_n\} \) by computing

\[
z_n = \frac{x_n}{s}, \quad n = 1, 2, \ldots, N
\]

Since this set of data was taken from an actual run, it is necessary to remove the trend in the data. The most common technique for trend removal is to fit a low-order polynomial to the data using the least squares procedures (figure 21).\(^9\) The linear regression line with an intercept of \( b_0 \) and a slope of \( b_1 \) should then be subtracted from the original data values \( \{u_n\} \) as shown in figure 22.
Figure 21. Typical Data Scatter.
Figure 22. Trend Removal.

In the attempt to determine the precision of the experimental measurements, the probability distribution of $z_n$ is plotted (figure 23). By comparing the distribution with the normal error distribution, it becomes clear that the measurement was in fact under control.\textsuperscript{10} Also, the rejection of data $x_n > 3\sigma_n$ is consistent.
5.6 Uncertainty Analysis

The method of estimating uncertainty in this experimental data follows that of Kline and McClintock\textsuperscript{11}. The method is based on a careful specification of the uncertainties in various primary experimental measurements. When measurements are made and these measurements are later used to calculate some desired results of the experiments, the result is a given function of independent variables $x_1, x_2, x_3, \ldots, x_n$. Thus $R = R(x_1, x_2, x_3, \ldots, x_n)$. 
Let \( W_r \) be the uncertainty in the result and \( w_1, w_2, \ldots, w_n \) be the uncertainties in the independent variables. The uncertainty in the result is

\[
w_r = \left[ \left( \frac{\partial R}{\partial x_1} w_1 \right)^2 + \left( \frac{\partial R}{\partial x_2} w_2 \right)^2 + \cdots + \left( \frac{\partial R}{\partial x_n} w_n \right)^2 \right]^{1/2}
\]

To numerically evaluate the partial derivatives, consider perturbing the independent variables \( x_1, x_2, x_3, \ldots, x_n \) by \( \Delta x_1, \Delta x_2, \ldots, \Delta x_n \). For small values of \( \Delta x \) the partial derivatives can be well approximated by

\[
\frac{\partial R}{\partial x_1} \approx \frac{R(x_1 + \Delta x_1) - R(x_1)}{\Delta x_1}
\]

\[
\frac{\partial R}{\partial x_2} \approx \frac{R(x_2 + \Delta x_2) - R(x_2)}{\Delta x_2}
\]

Calculations for the run mentioned above indicates an uncertainty in the calculation of bulk enthalpy is \( \pm 197.9 \) kJ/kg or \( \pm 5.7 \% \). A plot of arc current versus bulk enthalpy with the calculated error bars is shown in figure 24.
Figure 24. Arc Current Vs Bulk Enthalpy with Error Bars.

Detailed calculation is available in the appendix A.
CONCLUSIONS AND RECOMMENDATIONS

A continuous flow arc heated test facility has been put into operation at UT Arlington, and its performance map has been determined by the energy balance method. The energy balance method for performance calculation is relatively inexpensive and simple, however, the method gives no indication of the enthalpy distribution at the exit plane. Results indicate that the development tests conducted at AEDC can be replicated. The bulk enthalpy is nearly linear with increasing arc current and arc power within the operating envelope. Plenum chamber pressure is heavily influenced by the injection mass flow rate. Error analysis shows that the instrumentation installed yield an acceptable amount of accuracy. The results were different from previous numerical code calculations, but revealed the same trends. During this study, there were many obstacles such as electrical noise, constricted mass flow problems, ignition problems, and many others. Most of these problems have been solved except the ignition problem. The difficulty igniting the arc can be overcome by modifying the ignition process. Further studies should include varying the nozzle throat size, nozzle geometry, and test at higher mass flow rates. It may also be informative to perform enthalpy measurements by other methods such as using a water-cooled total enthalpy probe.
APPENDIX A

UNCERTAINTY ANALYSIS
Measurements taken during the experiment along with the transducer description and accuracy is shown in the following table below.

**Table 4. Instrument Accuracies**

<table>
<thead>
<tr>
<th>Measurements</th>
<th>Units</th>
<th>Transducer</th>
<th>Manufacturer Specs.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cooling Inlet Manifold</td>
<td>MV</td>
<td>Thermocouple T</td>
<td>± 1 °C</td>
</tr>
<tr>
<td>Anode Cooling Outlet</td>
<td>MV</td>
<td>Thermocouple T</td>
<td>± 1 °C</td>
</tr>
<tr>
<td>Cathode Cooling Outlet</td>
<td>MV</td>
<td>Thermocouple T</td>
<td>± 1 °C</td>
</tr>
<tr>
<td>Critical Flow Venturi</td>
<td>MV</td>
<td>Thermocouple T</td>
<td>± 1 °C</td>
</tr>
<tr>
<td>Temperature</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Critical Flow Venturi Pressure</td>
<td>MV</td>
<td>Omega</td>
<td>± 0.4% FS (1000)</td>
</tr>
<tr>
<td>Critical Flow Venturi Coefficient</td>
<td>-</td>
<td>FlowDyne</td>
<td>± 0.75%</td>
</tr>
<tr>
<td>Anode Cooling Flow Rate</td>
<td>Hz</td>
<td>Sponsler</td>
<td>± 0.3% FS</td>
</tr>
<tr>
<td>Cathode Cooling Flow Rate</td>
<td>Hz</td>
<td>Sponsler</td>
<td>± 0.3% FS</td>
</tr>
<tr>
<td>Arc Current</td>
<td>Volts</td>
<td>Halmar Control Board</td>
<td>± 0.25% FS</td>
</tr>
<tr>
<td>Arc Voltage</td>
<td>Volts</td>
<td>Halmar Control Board</td>
<td>± 0.25% FS</td>
</tr>
</tbody>
</table>

Taking a time slice of run MAR07-03 and resolving the raw data to their respective unit yields the following data table:
Table 5. Experiment Data

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Anode Temperature Rise</td>
<td>3.13E-04</td>
<td>mV</td>
</tr>
<tr>
<td>Cathode Temperature Rise</td>
<td>2.54E-04</td>
<td>mV</td>
</tr>
<tr>
<td>Critical Flow Pressure</td>
<td>5.35E-02</td>
<td>mV</td>
</tr>
<tr>
<td>Critical Flow Temperature</td>
<td>1.95E-05</td>
<td>mV</td>
</tr>
<tr>
<td>Anode Cooling Flow Rate</td>
<td>218</td>
<td>Hz</td>
</tr>
<tr>
<td>Cathode Cooling Flow Rate</td>
<td>180</td>
<td>Hz</td>
</tr>
<tr>
<td>Arc Current</td>
<td>381.1</td>
<td>Amp</td>
</tr>
<tr>
<td>Arc Voltage</td>
<td>1960.33</td>
<td>Volts</td>
</tr>
</tbody>
</table>

From the energy balance equation

\[
h_r = \frac{1}{\dot{m}_g} \left[ IV + (\dot{m}C_p T)_{g in} - (\dot{m}C_p (T_{out} - T_{in}))_A - (\dot{m}C_p (T_{out} - T_{in}))_C \right]
\]

the uncertainty for individual parameter is calculated.

Gas mass flow rate, \( \dot{m}_g \).

Given

\[ T_g = 1.95E-05 \text{ mV} \approx 273.76 \pm 1 \text{ C}^\circ \text{ or } 492.77 \pm 1.8 \text{ R}^\circ \]

\[ P_{Crit} = 5.35E-02 \text{ mV} \approx 535 \pm 4 \text{ PSI} \]

\[ K_{Crit} = 0.791 \pm 0.0059325 \]

Let

\[ \Delta T_{Crit} = 1 \text{ R}^\circ \]
\[ \Delta P_{\text{Crit}} = 1 \text{ PSI} \]

\[ \Delta K_{\text{Crit}} = 0.001 \]

then

\[ \dot{m}_{\text{e}} = \frac{K \dot{P}_{\text{Crit}}}{\sqrt{T_{\text{Crit}}} \sqrt{492.77}} \]

\[ = \frac{0.791 \cdot 535}{19.06 \text{ lb/min}} \]

we have

\[ \frac{\partial \dot{m}_{\text{e}}}{\partial K} = \frac{19.09 - 19.06}{0.001} = 30 \]

\[ \frac{\partial \dot{m}_{\text{e}}}{\partial P_{\text{Crit}}} = \frac{19.10 - 19.06}{1} = 0.039 \]

\[ \frac{\partial \dot{m}_{\text{e}}}{\partial T_{\text{Crit}}} = \frac{19.04 - 19.06}{1} = -0.016 \]

\[ \therefore \omega_{\text{e}} = \left[ 30^2 \cdot 0.0059325^2 + 0.039^2 \cdot 4^2 + (-0.016)^2 \cdot 1.8^2 \right]^{\frac{1}{2}} = 0.238 \text{ lb/min} \]

or \[ \omega_{\text{e}} = 1.802 \times 10^{-3} \text{ kg/sec} \]

Arc power, \( IV \).

Given

\[ I = 381.1 \pm 4 \text{ Amp} \]

\[ V = 1960.3 \pm 6.625 \text{ Volts} \]

Let

\[ \Delta I = 1 \text{ Amp} \]

\[ \Delta V = 1 \text{ Volt} \]

we have
\[ w_p = 8239.2 \text{ Watts} \]

1.1.1. Cooling Water Flow Rates.

\[ \dot{m}_A = 218 \frac{\text{Cycle}}{s} \cdot \frac{1}{150.8 \text{ Cycle}} \cdot 3.7854 \frac{\text{kg}}{\text{gal}} = 5.4722 \frac{\text{kg}}{s} \]

\[ \dot{m}_C = 180 \frac{\text{Cycle}}{s} \cdot \frac{1}{192.8 \text{ Cycle}} \cdot 3.7854 \frac{\text{kg}}{\text{gal}} = 3.534 \frac{\text{kg}}{s} \]

\[ w_{\dot{m}_A} = 0.0346 \text{ kg/sec} \]

\[ w_{\dot{m}_C} = 0.0266 \text{ kg/sec} \]

Solving for the uncertainty in the calculation of the bulk enthalpy

Given

\[ T_g = 273.76 \pm 1 \text{ K} \]

\[ \Delta T_A = 8.29 \pm 1 \text{ K} \]

\[ \Delta T_C = 6.66 \pm 1 \text{ K} \]

\[ C_p \text{ N}_2 = 1.0416 \text{ kJ/kg - K} \]

\[ C_p \text{ H}_2\text{O} = 4.184 \text{ kJ/kg - K} \]

\[ \dot{m}_g = 0.1441 \pm 0.001802 \text{ kg/s} \]

Power, IV = 747.082 ± 8.2392 kW

\[ \dot{m}_A = 5.4722 \pm 0.0346 \text{ kg/s} \]

\[ \dot{m}_C = 3.534 \pm 0.0266 \text{ kg/s} \]

Let

\[ \Delta \dot{m}_g = 0.001 \text{ kg/s} \]

\[ \Delta \text{IV} = 1 \text{ kW} \]
\[ \Delta T_A = 0.1 \text{ K} \]
\[ \Delta T_C = 0.1 \text{ K} \]
\[ \Delta \dot{m}_A = 0.01 \text{ kg/s} \]
\[ \Delta \dot{m}_C = 0.01 \text{ kg/s} \]

Then

\[ h_T = 3468.8467 \text{ kJ/kg} \]

\[ \frac{\partial h_T}{\partial \dot{m}_g} = \frac{|3446.905 - 3468.847|}{0.01} = 2194.141 \]

\[ \frac{\partial h_T}{\partial \dot{m}_{IV}} = \frac{3475.786 - 3468.847}{1} = 6.939 \]

\[ \frac{\partial h_T}{\partial \Delta T_A} = \frac{|3309.959 - 3468.847|}{1} = 158.888 \]

\[ \frac{\partial h_T}{\partial \Delta T_C} = \frac{|3366.236 - 3468.847|}{1} = 102.611 \]

\[ \frac{\partial h_T}{\partial T_g} = \frac{3469.888 - 3468.847}{1} = 1.041 \]

\[ \frac{\partial h_T}{\partial \dot{m}_A} = \frac{|3466.440 - 3468.847|}{0.01} = 240.7 \]

\[ \frac{\partial h_T}{\partial \dot{m}_C} = \frac{|3466.913 - 3468.847|}{0.01} = 193.4 \]
Solving,

\[ w_{hr} = \left[ 2194.141^2 \cdot 0.001802^2 + 6.939^2 \cdot 8.2392^2 + 158.888^2 \cdot 1^2 + 102.611^2 \cdot 1^2 + 1.041^2 \cdot 1^2 \right]^{0.5} \\
+ 240.7^2 \cdot 0.0346^2 + 193.4^2 \cdot 0.0266^2 \]

\[ w_{hr} = 197.878 \ \frac{kJ}{kg} \text{ or } 5.704 \% \]
APPENDIX B

EMPIRICAL SCALING RELATIONS
From figure 16 through figure 18, it is reasonable to assume that the plenum pressure is the linear combination of arc current and injection mass flow rate. Let the equation
\[ P_r = a(I) + b(\dot{m}_g) + c \]
represent the line. Then it is possible to solve for a, b, and c where
\[ \sum_{n=1}^{N_{Scan}} (P_{r,Exp} - P_{r,Emp})^2 \text{ is minimum} \]
Solving for the linear coefficients yields:
\[ a = 0.002351 \]
\[ b = 32.14014 \]
\[ c = -1.21632 \]
The same conclusion can be arrived for the bulk enthalpy where it is also a linear combination of arc current and mass flow rate, \( H_r = a(I) + b(\dot{m}_g) + c \). Solving yields
\[ a = 4.71518 \]
\[ b = -5818.91663 \]
\[ c = 2799.42544 \]
With the two linear equations, it is then possible to plot lines of constant arc current and mass flow rate as shown earlier. The carpet plot for electrical power versus bulk enthalpy can be obtained using the same method. Assume
\[ P_r = a(P) + b(\dot{m}_g) + c \text{ yields} \]
\[ a = 0.00152 \]
\[ b = 22.70456 \]
\[
c = -0.32064
\]

and

\[
H_r = a(P) + b(\dot{m}_g) + c \quad \text{yields}
\]

\[
a = 2.80249
\]

\[
b = -22952.77277
\]

\[
c = 4651.65521
\]
REFERENCES


