ON THE NEAR FIELD MEAN FLOW STRUCTURE OF TRANSVERSE JETS
ISSUING INTO A SUPersonic FREESTREAM

by

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To my parents who have given me their constant support throughout my academic career and this research project, I cannot describe nor repay the strength they have given me to reach this level of academic achievement.

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ABSTRACT

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The near field mean flow structure of transverse jets issuing from a surface into supersonic crossflow is examined using numerical methods and separation topology. The Navier-Stokes solver Falcon, developed at Lockheed Martin, was used to simulate the interaction between the jet and freestream over a flat plate and a generic missile body. The near field flow structure included a $\lambda$ bow shock upstream of the jet interacting with the approaching boundary layer that forms a pair of horseshoe vortices while another $\lambda$-structure closer to the jet formed a second pair of horseshoe vortices. As the jet was turned downstream by the crossflow, the so-called barrel shock terminates in a Mach disk while vortices formed within the jet plume. Downstream of the jet exit, new flow structure was identified in the form of three pairs of vortices. Horn, near field and far
field wake vortices were present downstream of the jet as well as a series of compression waves resulting in a gradual pressure rise downstream of the jet overexpansion. The wave formations and the vortices formed from them affected separation topology, performance parameters and amplification coefficients. The current understanding of the flow structure in the near field of a transverse jet in supersonic flow must be amended to include these newly identified vortices and compression waves.
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NOMENCLATURE

\( a \) = speed of sound
\( A \) = area
\( B_1 \) = empirical constant (19.25) (Smith 1990)
\( C_{b1} \) = empirical constant (Spalart-Allmaras 1994)
\( C_{b2} \) = empirical constant (Spalart-Allmaras 1994)
\( C_f \) = local skin friction coefficient = \( \frac{\tau_w}{q_w} \)
\( C_{w1} \) = empirical constants (Spalart-Allmaras 1994)
\( d \) = distance to the wall (Spalart-Allmaras 1994)
\( d_{\text{jet}} \) = jet diameter (0.1 in (2.54 mm) FP - 0.342 in (8.69 mm) BOR)
\( \tilde{e} \) = mass-averaged internal energy
\( E_1 \) = empirical constant (1.70) (Smith 1990)
\( E_2 \) = empirical constant (0.70) (Smith 1990)
\( f_w \) = empirical function (Spalart-Allmaras 1994)
\( f_{nw} \) = empirical function (Smith 1990)
\( f_{\nu 1} \) = empirical function (Spalart-Allmaras 1994)
\( F \) = force
\( F_1 \) = blending function (Menter 1993)
\( \tilde{H} \) = mass-averaged total enthalpy
\( J \) = momentum flux ratio = \( \frac{\gamma}{\gamma - 1} P_{\text{exj}} M_{\text{exj}}^2 / \gamma \rho_{\infty} M_{\infty}^2 \)
\( k \) = turbulent kinetic energy
\( l \) = turbulent length scale
\( L \) = wall proximity function (Smith 1990)
\( L_{\text{ref}} \) = reference length (18 in (457.2 mm) FP - 1.67 in (42.4 mm) BOR)
\( \dot{m} \) = mass flow
\( M \) = Mach Number
\( \hat{n} \) = unit normal
\( p \) = pressure
\( \bar{p} \) = time averaged pressure
\( q \) = dynamic pressure = \( \frac{\gamma}{2} \bar{p}_{\infty} M_{\infty}^2 \)
\( q_{tij} \) = heat sources and/or sinks
\( R \) = gas constant
\( Re \) = Reynolds Number = \( \frac{V_{\infty} L_{\text{ref}}}{\nu}_{\infty} \)
\( s \) = entropy
\( S_a \) = area vector of a cell face
\( S_q \) = empirical constant (0.20) (Smith 1990)
\( T \) = thrust
\(\tilde{T}\) = mass average temperature

\((u,v,w)\) = \((x,y,z)\) velocity components

\(\tilde{u}\) = mass-average velocity

\(V\) = velocity

\(t\) = time

\((x,y,z)\) = Cartesian coordinates

\(\frac{\partial \tilde{p}^*}{\partial x^*}\) = non-dimensional pressure gradient in the x direction

\(\alpha\) = empirical constant \((13/25)\) (Wilcox 1998)

\(\beta^*\) = empirical function (Wilcox 1998)

\(\chi\) = kinematic viscosity ratio

\(\delta\) = boundary layer thickness

\(\delta_{ij}\) = Kronecker delta

\(\eta\) = Van Driest parameter

\(\phi\) = accuracy switch \((0=\text{first order}, 1=\text{high order})\)

\(\gamma\) = specific heat ratio

\(\kappa\) = numerical scheme switch

\(\mu\) = absolute viscosity

\(\mu_t\) = turbulent viscosity

\(\nu\) = kinematic viscosity

\(\tilde{\nu}\) = mass-averaged kinematic eddy viscosity

\(\omega\) = specific dissipation rate

\(\bar{\rho}\) = time-average density

\(\sigma\) = turbulent Prandtl number (Spalart-Allmaras 1994)

\(\sigma^*\) = empirical constant \((0.5)\) (Wilcox 1998)

\(\sigma_k\) = \(0.5\cdot F_1 + (1-F_1)\) (Menter 1993)

\(\sigma_{\omega}\) = \(0.5\cdot F_1 + \sigma_{\omega 2}\cdot (1-F_1)\) (Menter 1993)

\(\sigma_{\omega 2}\) = empirical constant \((0.856)\) (Menter 1993)

\(\tilde{\tau}_{ji}\) = mass-averaged shear stresses

\(\tau'_{ji}\) = shear stress fluctuations

\(\rho u_{ji}\overline{u_i'}\) = Reynolds stresses

\(\rho u_{ji}'h'\) = turbulent heat conduction

\(\nabla\) = forward difference operator

\(\Delta\) = backward difference operator

subscripts

\(bc\) = boundary condition
\(exj\) = exit of jet
\(i\) = index
\(int\) = interior
\( j \) = index or jet
\( \text{jet} \) = jet
\( ji \) = indices or jet interaction
\( \text{obs} \) = obstruction
\( \text{plate} \) = plate
\( \text{ref} \) = reference
\( t \) = total
\( t_{\text{jet}} \) = total of jet
\( t_{\infty} \) = freestream total
\( \text{wash} \) = washout
\( x \) = x component
\( y \) = y component
\( z \) = z component
\( \infty \) = ambient

**Acronyms**

- **BOR** = Body of Revolution
- **CFD** = Computational Fluid Dynamics
- **FP** = Flat Plate
- **MUSCL** = Monotonic Upstream Scheme for Conservation Laws
- **RHS** = Right Hand Side
- **RJCS** = Reaction Jet Control Systems
- **PDEs** = Partial Differential Equations
- **SCRAMJET** = Supersonic Combustion Ramjet
- **TJISF** = Transverse Jet In Supersonic Flow
- **TVC** = Thrust Vector Control
- **V/STOL** = Vertical/Short Take Off and Landing
CHAPTER 1

INTRODUCTION

Although this dissertation was motivated by reaction jet control systems (RJCS) operating transverse to a missile body in supersonic flow, much of the past research concerning transverse jets was performed on flat plates. Moreover, the counter-rotating vortices that dominate the jet plume were first identified in the subsonic work of Fric and Roshko (1989) for a flat plate. For these reasons, this research began by examining transverse jets at low pressure ratio (PR) issuing from a flat plate into subsonic flow and built up to high PR jets issuing from a missile body into supersonic flow.

In this chapter, background concerning the development of our understanding of the near field flow structure is given followed by a review of previous work in chapter 2. Chapter 3 describes the motivation, scope and approach for this research program followed by a validation of the flow models in chapter 4. Chapter 5 discusses the results of the numerical analyses beginning with a subsonic freestream with a laminar approaching boundary layer and jet PR 1.5 over a flat plate and ending with a supersonic freestream with a turbulent boundary layer and jet PR 1000 over a missile body. Chapter 6 completes the dissertation by stating conclusions and outlining future work required to further our understanding of the near field flow structure around transverse jets.

Motivations for the study of transverse jets date back more than 50 years. Research directed toward secondary fluid injection conducted by Morkovin et al. (1952)
was motivated by the desire to steer vehicles in endo-atmospheric flight. Since then, jets have been considered for a number of aerospace applications such as the hover capability for vertical/short take-off (V/STOL) aircraft, fuel injection in supersonic combustion ramjets (SCRAMJETS) and cooling rocket nozzle walls. These applications covered a wide range of freestream and jet conditions from quiescent environments with perfectly expanded and underexpanded jets in V/STOL vehicles to hypersonic freestreams with underexpanded jets in interceptors. The present study is directed at supersonic freestreams with underexpanded jets issuing normal to the surface for application to reaction jet control systems (RJCS) for flight-path control. Although transverse jets in supersonic crossflows have been studied for more than 60 years, the understanding of the near-field flow structure is still not complete. Much of the past research was conducted to quantify performance effects of a particular RJCS. Performance parameters may have included forces and moments for a RJCS or jet mixing and penetration for a SCRAMJET fuel injection system. Few studies have focused on the near-field mean flow structure. Those that have were concerned principally with the upstream flow structure. Downstream flow structure of the near field has received much less attention and is therefore not as well understood. Although most systems involving transverse jets have been successfully employed, their success has been based on global performance data gathered experimentally with little consideration of the flow field details. A better understanding of these details could provide better control over transverse jet systems and their applications and may lead to better, more innovative approaches to employing these systems in future applications.
Morkovin et al. (1952) was an early study of transverse jets issuing from both a flat plate and a body of revolution into a supersonic freestream. This work provided a qualitative description of near field flow and gave some guidance on how to treat jet interaction problems. Morkovin et al. flow visualizations and surface pressure data showed several flow structures in the near field. From the flow visualization and the limited centerline surface pressure measurements, an upstream bow shock, a separation shock, the barrel shock around the jet and a Mach disk were clearly identified. Figure 1.1 illustrates the identified flow structures on the cone-cylinder test article.

![Flow Structures Diagram](image)

**Figure 1.1** Morkovin’s jet interaction flow structure.

The flow in Figure 1.1 is from left to right. The cone-cylinder model was mounted on a sting shown on the right side of the figure. The first shock, identified as he “nose shock” in the figure, is nowadays known as the bow shock. This shock is reflected off the tunnel wall and runs downstream. The boundary layer building from the nose tip remains relatively thin until the cylindrical portion of the body. At this point, an expansion fan occurs at the base of the cone. The obstruction of the jet causes a second
bow shock to form, interacting with the approaching boundary layer. This generates a high pressure region in front of the jet. A dashed line is shown to indicate the jet plume boundary and how the jet is bent downstream. As the jet is bent downstream, a shock around the jet, terminated by a barrel shock, is formed. Downstream of the barrel shock, a mixing region was identified as well as a reflection shock from the Mach disk running toward the surface. This shock reflects off the cylindrical surface and continues running downstream. On both the upper and lower surfaces of the cylinder, a shock identified as the “jet shock window interaction” is shown. A tunnel shock, of unknown origin, is shown in the upper left side of the figure. This shock appears to miss the body as it travels downstream. An envelope shock is shown in the upper right side of the figure. It appears to originate from the jet, but is drawn as weak compressions that eventually coalesce into a shock. Although much of the flow structure currently attributed to transverse jets in a supersonic freestream was identified by Morkovin et al. it is evident that understanding of the near field was mostly qualitative.

Cubbison et al. (1961) obtained more detailed surface pressure distributions from 91 pressure taps and along with flow visualization clarified and further identified the flow structure in the near field as shown in Figure 1.2. Features include a leading edge shock, an upstream separation, an upstream stagnation point and a bow shock. Although the near-field, mean flow structure was still incomplete, sufficient understanding of the complexities led researchers to investigate the effect of certain parameters, such as the momentum flux ratio, on the performance of transverse jets rather than continue
examine the near field. This left the near-field flow structure incomplete for more than 30 years.

Figure 1.2 Cubbison’s flow structure.

Gruber et al. (1996) noted a pair of vortices in the jet plume in their study of transverse jets in supersonic crossflow and attributed them to the same phenomena noticed by Fric and Roshko (1989) who conducted studies of transverse jets issuing into a subsonic freestream. Fric and Roshko’s smoke visualization revealed a pair of vortices within the mean flow of the jet as it was turned downstream, the so-called counter-rotating jet vortices, as shown in Figure 1.3.
Figure 1.3 Fric and Roshko’s flow structure.

Fric and Roshko concluded that the vorticity from the boundary layer built up on the jet nozzle wall was the source of these vortices. Fric and Roshko also showed two sets of unsteady vortices and another set of steady vortices, referred to as horseshoe vortices. There is a set of unsteady vortices in the wake of the jet similar to the vortices seen aft of a cylinder in crossflow but with different character, and a set of vortices on the windward side of the boundary between the jet and the freestream, referred to as shear layer ring vortices. The ring and wake vortices are unsteady. The jet vortices in the jet plume were identified as steady and shown by Gruber et al. to dominate the downstream flow field.
Roger and Chan (1993) identified a second pair of horseshoe vortices originating upstream of the jet from CFD results. They found that the additional pair of vortices originated between the jet and the first pair of horseshoe vortices and convected around the jet away from the surface. Adding this to the previous flow structure and the illustrations from Champigny and Lacau (1994) and Gruber et al. (1995), Figure 1.4 shows the near field mean flow structure of a transverse jet in supersonic freestream as it is understood today. The freestream in this figure moves from right to left. The jet issues normal to the freestream direction. A three-dimensional bow shock is formed by the obstruction of the jet to the freestream. This bow shock interacts with the approaching boundary layer (not shown) and develops into a three-dimensional, $\lambda$-shock system (Shapiro 1954). The separation associated with the $\lambda$-shock system creates a transverse pressure gradient which stretches the separation around the jet creating vortices that
convect downstream. These vortices are named horseshoe vortices because the path they trace along the surface is shaped like a horseshoe. Aft of the $\lambda$-shock system, the freestream and jet interact, turning the jet downstream and forming a shock around the jet. This shock is referred to as the barrel shock which terminates in a Mach disk. Within the jet, counter-rotating vortices are formed from the vorticity in the boundary layer of the jet. Aft of the barrel shock on the surface, a separation bubble occurs. As this separation bubble closes, a three-dimensional reattachment shock is formed. From Figure 1.4, the complexity of the near field of transverse jets in supersonic flow is clear. Although much is known about this flow field, the understanding of this complex structure is still not complete. The importance of understanding this flow field lies in the effect these structure have on global performance parameters such as normal force and pitching moment.

The flow structure produced by the presence of the jet modifies the pressure field on the body. The integration of the difference in pressure fields on a typical missile body between the jet on and jet off conditions gives a force referred to as the jet interaction force. The jet interaction force can be broken down into two components; the obstruction component and the washout component. The obstruction component is the integrated pressure field difference just on the body, not including any aerodynamic surfaces such as fins, strakes, control surfaces or other lifting surfaces. Aerodynamic surfaces downstream of the jet experience an interference effect due to the wake from the jet interaction. This interference effect modifies the pressure field on the aerodynamic surfaces. Integration of the difference in the pressure fields on these surfaces between the
jet on and jet off conditions give the washout component of the jet interaction force. The sum of the obstruction and washout components is the total jet interaction force, $F_{ji}$, (Schroeder 1999) experienced as a result of the flow structure produced by the presence of the jet, that is,

$$F_{ji} = F_{obs} + F_{wash}$$

The importance of this force cannot be understated as it can be of the same magnitude as the jet thrust,

$$T_j = \dot{m}_j V_{e sj} + (p_{esj} - p_w) A_{esj}$$

At certain flow conditions, Mahmud and Bowersox (2005) showed that the jet interaction force can amplify the jet thrust by over 100% while at other conditions the jet can be completely nullify the thrust. This large swing in magnitude and direction of the jet interaction force can cause complications in autopilot design if not properly understood and quantified.

The complex jet interaction flow field has challenged researchers for decades. Although a significant amount of research has been dedicated to transverse jets, basic understanding of the downstream flow structure has been left virtually uninvestigated and experimental and numerical techniques have not provided adequate resolution to identify significant flow features such as the origin of the horseshoe vortices. However, recent developments in numerical simulation have provided means to advance the understanding of transverse jets in a supersonic freestream. In the past, both mean and instantaneous flow phenomena have been more readily identified experimentally, but advances in numerical techniques and computer processing power provide an advantage over
experimental techniques and previous numerical methods. The growth of computer processing speed has allowed grid resolutions to become finer to the point of outpacing the resolution capability of today’s experimental techniques. This advantage allows a more detailed examination of thin boundary layers, shock structure and flow field separation. It also provides the ability to collect flow field data in areas of the flow where instrumentation cannot fit without disrupting the flow field. The current study makes use of these advantages and applies numerical simulation to the analysis of the near field mean flow structure of transverse jets in a supersonic freestream.
CHAPTER 2
LITERATURE REVIEW

2.1 Analytical Studies

An early attempt by Ferrari (1959) at determining the interference forces due to jet interaction used an equivalent body to represent the transverse jet. The shape of the equivalent body was determined by finding the centerline of the jet and assuming the jet cross-sectional shape remained similar along the centerline. This analysis ignored viscosity, mixing between the jet and the freestream and used detached shock theory to determine that the shape of the upstream bow shock. Ferrari noted that, in practical missile design, jets were placed at the base of body. Such jet placement meant only the upstream flow field required examination. The equivalent body representation compared adequately with force data from Liepmann (1958) and Amick, et al. (1963).

Schetz and Billig (1966) assumed that jet penetration into a supersonic crossflow was a two-stage process: a penetration stage and a coaxial turbulent mixing stage. Their model assumed an equivalent solid body for the jet as well. The centerline of the jet was bent downstream by the freestream interaction with the jet. The cross-section of the jet was distorted by pressure differences on the front and back faces of the equivalent body. The concept of “effective” back pressure was introduced to approximate the location of the Mach disk of the jet. Schetz and Billig also pointed out the importance of the momentum flux ratio on the jet penetration height.
Billig, et al. (1971) applied the conservation equations to a coupled dual control volume approach to transverse jet in supersonic crossflow. A boundary between a control volume for the freestream and a control volume for the jet was shared to couple the equations and the concept of “effective” back pressure was used, but modified to 2/3 of the ambient total pressure, as a boundary condition to the conservation equations. The focus of this work was the determination of the jet centerline trajectory and the Mach disk location to further the understanding of the mass distribution in the interaction region. Billig, et al. made this comparison for a sonic nozzle at a total pressure ratio of 1.6 and a freestream Mach number of 2.7. The results compared well with the hydrogen injection experiments of Orth, et al. (1969) to five jet diameters downstream and adequately to ten diameters downstream.

Heister and Karagozian (1990) assumed the jet to be a vortex pair and used two-dimensional potential flow theory to find the jet orientation. With the jet orientation, the mass and momentum equations for the jet and a force balance on the jet cross section were used to find the jet velocity as a function of time. This result was integrated to obtain the jet centerline trajectory and, thus, jet penetration height. The results of this rather involved theory compared well with the jet centerline trajectories of Orth, et al (1969). Although Heister and Karagozian’s study focused on thrust vector control (TVC), no surface pressure data in the near field was presented.
2.2 Numerical Studies

Margason (1993) observed, in his review of transverse jets in supersonic flow (TJISF), that all computational investigations up to that time inadequately resolved the near field mean flow. Grid density and turbulence models were both identified as areas requiring improvement. Moreover, Dormieux and Marsaa-Poey (1993) compared experimental data for a supersonic jet issuing into a Mach 0.8 crossflow with numerical calculations from both an Euler and a Navier-Stokes solver. This comparison showed that the Euler solver could not properly predict the vortices near the wall downstream of the jet exit and neither solver could match experimental pressure distributions. Dormieux and Marsaa-Poey concluded that the resolution of the near field mean flow was not adequate to properly predict the pressure distribution.

Shang, et al. (1989) also compared experimental and numerical data, but for a jet issuing into a Mach 12 crossflow. The numerical results compared well with the experimental pressure data and showed the upstream \( \lambda \)-shock system. However, the Mach disk was not present and the numerical results revealed four upstream separation zones which were not found experimentally. Shang, et al. noted that the numerical results suffered because of grid density and turbulence modeling and concluded that the absence of the Mach disk was a consequence of the hypersonic crossflow.

Chan, et al. (1993) examined TJISF over a missile body using a full Navier-Stokes solver with turbulence modeling. The results show the bow shock and the upstream separations but no barrel shock or Mach disk. No mention of the downstream flow structure was made for the body alone at zero angle of attack. However, for the
body-fin configuration at $\alpha = -9^\circ$, two vortices were mentioned, one near the wall that wrapped around the jet (the horseshoe vortices) and one from the jet plume (the jet vortices). Beyond that, no other flow structure was identified, most likely because of a lack of adequate grid resolution and quality of the turbulence model.

Min, et al. (2005) performed a computational study of supersonic flow around a missile body to investigate the effect of transverse jet flow conditions on normal force and pitching moment. A very brief discussion of the near field mean flow structure was given. These authors identified the bow shock, barrel shock, separation shock and secondary shock as well as one upstream separation zone, one downstream separation zone and the jet vortices. No mention of horseshoe vortices or their origin was made. Although a description of the near field was given, no significant examination of it was made.

2.3 Experimental Studies

Morkovin, et al. (1952) published the first experimental study on a supersonic transverse jet issuing into a supersonic flow. This study considered a cone-cylinder body as well as a flat plate. From the pressure data taken and the visualization techniques used, the flow structure was inferred as shown in Figure 1.1. Ignoring shock reflections from the wind tunnel walls and the bow shock from the cone-cylinder body, the near field structure showed the bow shock, the separation shock, a single upstream separation, the barrel shock, the Mach disk and a downstream reattachment shock. No mention of vortices was made. With these experimental data, Morkovin, et al. identified a significant amount of the flow structure for the transverse jet in supersonic crossflow.
Janos (1961) examined the forces produced by a transverse supersonic jet issuing from a flat plate into a Mach 2 flow. This study assumed the flow structure of Morkovin and did not examine it experimentally in any detail. The focus of this study was measurement of the interference forces for various nozzle pressure ratios. Only pressure data were taken and the jet interaction forces were found by integrating this pressure data without examination of the flow structure.

Cubbison, et al. (1961) investigated the effects of pressure ratio, freestream Mach number and Reynolds number on the surface pressure distribution on a flat plate from a transverse jet with a laminar approaching boundary layer. The authors were the first to identify two separations upstream of the jet and that one of them was the origin of the horseshoe vortices wrapping around the jet. In addition, an upstream stagnation point was identified with the bow shock. A downstream separation was also identified for the first time. The authors, however, did not identify the barrel shock, Mach disk or the upstream separation shock. The additional flow structure identified by Cubbision, et al added significantly to the understanding of transverse jets in supersonic crossflow.

Amick, et al. (1963) performed experiments using a sphere-cone-cylinder body with a transverse supersonic jet issuing into a freestream that ranged from quiescent to supersonic. The main focus of this study was to quantify the forces on the body in the presence of the jet. The effects of angle of attack, jet location and body length were documented. The only near field flow structures mentioned in this study were the bow shock, the separation shock and the reattachment shock. All of these were inferred from surface pressure data. No mention of any vortices was made. Although Amick, et al.
quantified the interference forces in the near field, little investigation into the flow structure was done.

Zukoski and Spaid (1964) used the penetration height of the jet as a scaling parameter to predict the force on the surface of a flat plate. The penetration height was determined to be a function of jet pressure ratio and freestream Mach number. The only flow structure phenomena discussed were the bow shock, separation shock and jet vortices. Although the flow structure was not the focus of this study, the effect of jet penetration height on the surface force was an illuminating relation.

Schetz, et al. (1967) experimentally investigated the internal flow structure of highly underexpanded transverse jets in a supersonic crossflow. The purpose of this study was to show the adequacy of the “effective” back pressure concept in properly locating the Mach disk. The idea was that if the Mach disk was properly located, the surface pressure and, thus, the force and moment, could be adequately predicted. In this study, the only flow structure discussed was the bow shock, barrel shock, Mach disk and one upstream separation. Although it was concluded that a back pressure of 80% of ambient adequately predicted the location of the Mach disk, no relationship between the Mach disk location and the jet interaction force was reported.

Dahlke (1969) conducted tests of a ogive-cylinder body with a sonic transverse jet in transonic crossflow. This study was focused on measuring the downstream vorticity and mapping vortex locations. There was no discussion of the upstream flow structure. The study identified a single pair of trailing vortices that were influenced by the jet chamber to freestream pressure ratio and freestream Mach number. Although no
investigation of the near-field, mean flow structure was made, Dahlke succeeded in identifying parameters influencing vortices generated by a transverse jet in transonic crossflow.

During the 1970s and 1980s, transverse jet research in subsonic crossflows became popular as the development of vertical/short take off aircraft was in vogue. Much of that research was focused on identifying vortical flows and their effects downstream. Fric and Roshko (1989) gave a complete understanding of the vortical structures present in transverse jets in subsonic crossflows. They identified two sets of steady vortices and two sets of unsteady vortices. The unsteady vortices were identified as shear layer ring vortices present along the jet-freestream interface and the wake vortices present downstream of the jet along the wall. The shear layer ring vortices originate from the shear layer between the jet and the freestream crossflow on the windward side of the jet. The wake vortices occur aft of the jet similar to the vortices seen aft of a cylinder in crossflow. The steady vortices were the horseshoe vortices which begin upstream and wrapped around the jet and the counter-rotating vortices in the jet plume. The horseshoe vortices originated in the separation zone upstream of the jet. The recirculating flow in the separation zone is carried around the jet and downstream by the freestream crossflow creating a horseshoe-shaped trajectory. The counter-rotating vortices in the jet originate upon the emergence of the jet from the surface. As the jet was turned downstream by the freestream, a radial pressure gradient was created in the jet. In the presence of this gradient, the vorticity built up in the boundary layer of the jet nozzle caused counter-rotating vortices to be formed in the jet. Moreover, this study
considered only laminar approaching boundary layers. The study of transverse jets in subsonic crossflows by Fric and Roshko revealed four sets of vortices, but the flow structure for this case is fundamentally different from the flow structure in supersonic crossflow.

Gruber, et al. (1995) studied sonic transverse jet injection into supersonic crossflow to obtain a more thorough understanding of the dominant features of near field mixing. They identified an upstream and a downstream recirculation, and the upstream recirculation as the origin of the horseshoe vortices. They recognized the unsteady ring vortices and wake vortices and the steady counter-rotating vortices in the jet plume. They also identified the bow shock, barrel shock and Mach disk, but there was no discussion of a downstream recompression or vortices. Although Gruber, et al. examined the near-field flow structure, they identified flow structures that were already documented. Their focus was flow field vorticity and the possibility of freestream/jet mixing in the hope of improving supersonic combustion processes.
CHAPTER 3
RESEARCH MOTIVATION AND APPROACH

Research focused on TJISF over the last 60 years has been motivated by several applications. V/STOL vehicles have occupied much of these transverse jet studies, but fuel injection in scramjets, rocket nozzle thrust vectoring and reaction control jet systems have also received attention. The current study is motivated by reaction control jet systems on missile bodies. Experience has shown the complex nature of the near field flow structure of a transverse sonic or supersonic jet issuing into a supersonic crossflow causes counterintuitive behavior. At certain flight conditions, the modification of the near-field pressure distribution can significantly amplify the jet thrust. At other flight conditions, the thrust is attenuated and at even other conditions, the direction of the thrust can be reversed. All of these conditions need to be identified to properly design an autopilot. More important, the near-field flow structure must be understood to properly understand the counterintuitive behavior of reaction control jet systems and possibly discover a means of mitigating or even controlling the behavior. The focus of the current study is to provide a more detailed understanding of the flow structure and the mechanisms giving rise to them.

The scope of the research is limited to considering the near-field, mean flow structure of transverse perfectly expanded and underexpanded jets in supersonic crossflow although subsonic crossflows are considered as well. Simulations on a flat
plate at various crossflow Mach numbers and jet pressure ratios and on a body of revolution at various jet pressure ratios for a particular crossflow Mach number are conducted to examine this flow structure as well as simulations to validate the CFD code with laminar and turbulent boundary layers over a flat plate as well as a transverse jet in supersonic crossflow over a flat plate.

The approach to examining the near-field mean flow of a transverse jet in supersonic flow (TJISF) is based on the theory that at every point in the flow mass, momentum and energy are conserved. The theory also assumes the gas is perfect, there are no body forces or mass sources and sinks. Given this theory, the Navier-Stokes equations were derived from the laws of conservation, simplified by the assumptions and the flow quantities averaged to account for turbulent fluctuations to produce a mathematical model that was approximated into finite difference equations to calculate flow properties at discrete points when proper boundary conditions and an appropriate initial condition are applied. The numerical calculation method used in this study was validated using existing experimental data, then applied to other flow conditions and geometries to investigate the near field mean flow structure of TJISF.

Two numerical calculation methods were initially screened: Falcon, a code developed at Lockheed Martin Corporation in Fort Worth, Texas, and GASP, developed by Aerosoft, Inc., in Blacksburg, Virginia. Both models were used to predict supersonic laminar flow over a flat plate. These predictions were compared to Van Driest’s (1956) compressible laminar boundary layer theory. Turbulence models were then validated using experimental data. Falcon with Smith’s (1990) and GASP with three different
turbulence models due to Wilcox (1998), Menter (1993) and Spalart and Allmaras (1994) were used to compute supersonic turbulent flow over a flat plate. The boundary layer profiles from both Falcon and GASP were compared with experimental data from Shutts, et al (1955) and Mabey, et al (1974). Good agreement would increase confidence in the codes ability to simulate more complex flows.

Following this validation, both codes were used to compute a TJISF over a flat plate. The results were compared with experimental data from Dowdy and Newton (1963). The intent was to verify that the codes were able to capture all of the flow features that produce the experimental pressure distribution. Flow conditions for all of these validation simulations are shown in Table 3.1. Good comparison with all of these cases would provide confidence that the codes would give reasonable and reliable data for the complex flow structure of TJISF. This validation also allowed the selection of a code for further study of TJISF.

Table 3.1 Flow Conditions For Validation Simulations

<table>
<thead>
<tr>
<th>Geometry</th>
<th>Boundary layer type</th>
<th>Crossflow Mach number</th>
<th>Reynolds number (millions)</th>
<th>Nozzle type</th>
<th>Nozzle pressure ratio</th>
<th>$J$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flat Plate</td>
<td>Laminar</td>
<td>2.00</td>
<td>0.1</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Flat Plate</td>
<td>Laminar</td>
<td>4.00</td>
<td>0.1</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Flat Plate</td>
<td>Turbulent</td>
<td>2.23</td>
<td>25.0</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Flat Plate</td>
<td>Turbulent</td>
<td>4.50</td>
<td>28.1</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Flat Plate With Jet</td>
<td>Turbulent</td>
<td>2.61</td>
<td>3.4</td>
<td>Convergent</td>
<td>307.5</td>
<td>23.85</td>
</tr>
</tbody>
</table>

Following proper validation, calculations of TJISF on a flat plate at various pressure ratios were done to examine the flow structure and identify trends in force and
moment amplification coefficients. In addition, calculations of TJISF over a body of revolution were done to examine the difference in flow structure between a flat plate and body of revolution for a given jet pressure ratio. And finally, calculations of TJISF with a convergent-divergent nozzle at various pressure ratios on the same body were done to examine the differences in flow structure for jet pressure ratio on a body of revolution. Flow conditions for all of these calculations are shown in Table 3.2.
<table>
<thead>
<tr>
<th>Geometry</th>
<th>Boundary layer type</th>
<th>Crossflow Mach number</th>
<th>Reynolds number (millions)</th>
<th>Nozzle type</th>
<th>Nozzle pressure ratio</th>
<th>$J$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flat Plate With Jet</td>
<td>Laminar</td>
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<td>0.1</td>
<td>Convergent</td>
<td>1.5</td>
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The approach described herein provided adequate data to properly draw conclusions about the impact of freestream Mach number, jet pressure ratio, body geometry and nozzle geometry on jet interaction forces and moments.
4.1 Falcon Model

4.1.1 Equations of Motion

The Falcon Navier-Stokes solver was developed and is maintained by Lockheed Martin Aeronautics in Fort Worth, Texas. Falcon is a three-dimensional full Navier-Stokes solver modeling the Reynolds-averaged conservation equations. The Reynolds-averaged conservation equations are derived by time averaging the density and pressure and (Favre) mass averaging the velocity components, enthalpy and temperature. The conservation equations and the equation of state, written in Cartesian tensor notation are as follows:

\[
\frac{\partial \bar{p}}{\partial t} + \frac{\partial}{\partial x_i}(\bar{\rho} \bar{u}_i) = 0
\]

\[
\frac{\partial}{\partial t} (\bar{\rho} \bar{u}_i) + \frac{\partial}{\partial x_j} (\bar{\rho} \bar{u}_j \bar{u}_i + \delta_{ij} \bar{p}) = \frac{\partial}{\partial x_j} \left[ \tau_{ij} - \rho u^* u_j^* \right]
\]

\[
\frac{\partial}{\partial t} \left( \bar{\rho} \bar{H} + \frac{\rho \bar{u}_j u_j}{2} - \bar{p} \right) + \frac{\partial}{\partial x_j} \left( \bar{\rho} \bar{u}_j \bar{H} + \bar{u}_j \frac{\rho \bar{u}_j u_j}{2} + \rho u^* u^*_j + q_{ij} - \bar{u}_i \left( \tau_{ij} - \rho u^* u^*_j \right) \right) =
\]

\[
+ \frac{\partial}{\partial x_j} \left( u_i \left( \tau_{ij} - \rho u^* u^*_j \right) \right)
\]

\[
\bar{p} = \bar{\rho} \bar{R} \bar{T}
\]
The turbulent fluctuation terms in the above equations are determined as follows,

\[
\frac{\rho u_{ij}^\sigma u_{ij}^\sigma}{2} = \mu_T \left( S_{ij} - \frac{1}{3} \frac{\partial \tilde{u}_i}{\partial x_k} \delta_{ij} \right) - \frac{2}{3} \tilde{p} k \delta_{ij}
\]

\[
\frac{\rho u_{ij}^\sigma u_{ij}^\sigma}{2} = \tilde{u}_j \tilde{p} k
\]

\[
\frac{\rho u_{ij}^\sigma h^\sigma}{2} = q_{ij} = \frac{\mu_T}{\Pr_T} \frac{\partial \tilde{h}}{\partial x_j}
\]

\[
u \left( \tau_{ij} - \frac{\rho u_{ij}^\sigma u_{ij}^\sigma}{2} \right) = \left( \mu + \mu_T \right) \frac{\partial k}{\partial x_j}
\]

Substituting into the conservation equations,

\[
\frac{\partial \tilde{p}}{\partial t} + \frac{\partial}{\partial x_i} \left( \tilde{p} \tilde{u}_i \right) = 0
\]

\[
\frac{\partial}{\partial t} (\tilde{p} \tilde{u}_i) + \frac{\partial}{\partial x_j} (\tilde{p} \tilde{u}_j + \delta_{ij} \tilde{p}) = \frac{\partial}{\partial x_j} \left[ \tilde{\tau}_{ij} - \left\{ 2 \mu_T \left( S_{ij} - \frac{1}{3} \frac{\partial \tilde{u}_i}{\partial x_k} \delta_{ij} \right) - \frac{2}{3} \tilde{p} k \delta_{ij} \right\} \right]
\]

\[
\frac{\partial}{\partial t} (\tilde{p} \tilde{H} + \tilde{p} k - \tilde{p}) + \frac{\partial}{\partial x_j} \left[ \tilde{p} \tilde{u}_j \tilde{H} + \tilde{u}_j \tilde{p} k + q_{ij} + q_{Lj} \right] =
\]

\[
\frac{\partial}{\partial x_j} \left[ \tilde{u}_j \left( \tilde{\tau}_{ij} - \left\{ 2 \mu_T \left( S_{ij} - \frac{1}{3} \frac{\partial \tilde{u}_i}{\partial x_k} \delta_{ij} \right) - \frac{2}{3} \tilde{p} k \delta_{ij} \right\} \right) \right] + \left\{ \mu + \frac{\mu_T}{\sigma_k} \right\} \frac{\partial k}{\partial x_j}
\]

\[
\tilde{p} = \rho R \tilde{T}
\]

The above equations are discretized using a first-order forward difference operator for the time derivative and a second-order central difference operator for the viscous terms. The MUSCL upwind extrapolation due to Van Leer (1976) is used for
determining the face properties. The upwind flux difference splitting method of Roe (1981) is used for the inviscid flux terms.

The flux at each cell face depends on properties of the cell volume. The cell properties are distributed to each cell face according to the Monotonic Upstream-centered Scheme for Conservation Laws (MUSCL) extrapolation method. Each primitive variable is extrapolated from the cell center to the cell face. For example, the density $\rho$ at the right cell face is

$$\bar{\rho}_{i+1/2} = \rho_i + \frac{\phi}{4} \left[(1 - \kappa_i) \nabla \rho_i + (1 + \kappa_i) \Delta \rho_i \right]$$

The left cell face is

$$\bar{\rho}_{i-1/2} = \rho_i - \frac{\phi}{4} \left[(1 + \kappa_i) \nabla \rho_i + (1 - \kappa_i) \Delta \rho_i \right]$$

where $\kappa_i = -1$ for an upwind scheme.

In order to reduce numerical oscillations, the fluxes at each cell face are then split into two parts to reflect the dependence domains according to characteristic theory using the flux-difference splitting method of Roe (1981). Once the equations are discretized in this form, the complete set of discretized equations can be solved for each cell through an iterative implicit solver to arrive at a steady-state solution.

4.1.2 Smith $\kappa$-$\omega$ Turbulence Model

Falcon uses a two-equation $\kappa$-$\omega$ turbulence model due to Smith (1990). This model is used to determine the turbulent viscosity and turbulent kinetic energy which
closes the system of equations. The following equations are used to solve the turbulent kinetic energy $k$ and the turbulent length scale $l$:

$$\frac{\partial (2pk)}{\partial t} + \tilde{u}_j \frac{\partial (2pk)}{\partial x_j} = 2f_{nw}(\tau_{ij} + \tau'_{ij}) \frac{\partial \tilde{u}_i}{\partial x_j} - \frac{2\tilde{p}(\sqrt{2k})^3}{B_1l} + \frac{\partial}{\partial x_j} \left[ S_q \tilde{p} \sqrt{2kl} \frac{\partial (2k)}{\partial x_j} \right]$$

$$\frac{\partial (2pkl)}{\partial t} + \tilde{u}_j \frac{\partial (2pkl)}{\partial x_j} = E_i l f_{nw}(\tau_{ij} + \tau'_{ij}) \frac{\partial \tilde{u}_i}{\partial x_j} - \tilde{p} \left( \frac{\sqrt{2k}}{B_1} \right)^3 \left( 1 + \left[ \frac{\partial}{\partial x_j} \left( E_z + S_q B_1 \left( \frac{\partial}{\partial x_i} \right)^2 \right) \right] \frac{l^2}{(k\lambda)^2} \right)$$

$$+ \frac{\partial}{\partial x_j} \left[ S_q \tilde{p} \sqrt{2kl} \frac{\partial (2kl)}{\partial x_j} \right]$$

The turbulent viscosity is related to the turbulent kinetic energy and the turbulent length scale by

$$\mu_i = \phi \frac{\tilde{p} \sqrt{2kl}}{B_1^{1/3}}$$

Refer to Smith (1990) for complete details of this model.

### 4.1.3 Boundary and Initial Conditions

For the no-jet, flat plate simulations, user-defined freestream conditions were used as the initial conditions. Figure 4.1 shows the boundary conditions used in Falcon.
Figure 4.1 Falcon boundary conditions for flat plate

The INFLOW boundary is the boundary where the flow enters the computational domain. The OUTFLOW boundary is where the flow leaves the computational domain. To the left and right of the INFLOW boundary, SYMMETRY boundary conditions were used. The SYMMETRY boundary assumes that the symmetry plane separates flow fields that mirror each other. In Figure 4.1, the SLIP boundary assumes a solid surface with no molecular adhesion of the flow. This means that the velocity at the surface is non-zero and tangent to the boundary. The NO-SLIP boundary assumes a solid surface with molecular adhesion. This means that the velocity is zero at the surface. The NO-SLIP boundary in Figure 4.1 simulates the flat plate area. The CHARACTERISTIC FAR FIELD boundary represents the flow far from the solid surface. Further details about the CHARACTERISTIC FAR FIELD boundary will be discussed later in this section.

At the inflow boundary, all of the primitive variables, namely, $\rho$, $\rho u$, $\rho v$, $\rho w$, $\rho e$, are specified. At the outflow boundary, none of the primitives were specified. The values of the primitives at the outflow boundary are copied directly from the first interior
point, the so-called zeroth-order extrapolation. The far-field boundary consists of four parts and is based on a one-dimensional approximation of the inviscid flow equations in characteristic form. Falcon determines if the flow is entering or leaving the boundary and whether it is subsonic or supersonic. For supersonic inflow, all of the primitive variables are taken from the user-defined freestream conditions. For supersonic outflow, all primitive variables are extrapolated from interior points by zeroth-order extrapolation. For subsonic inflow, the following equations are used to determine the primitive variables,

\[
\bar{p}_{bc} = \frac{1}{2} \left( \bar{p}_\infty + \bar{p}_{int} + \hat{n} \bar{p}_0 \tilde{a}_0 \left[ \frac{\Delta \tilde{V} \cdot S_s}{S_s} \right] \right)
\]

\[
\Delta \tilde{V} = (\tilde{u}_\infty - \tilde{u}_{int}, \tilde{v}_\infty - \tilde{v}_{int}, \tilde{w}_\infty - \tilde{w}_{int})
\]

\[
\bar{p}_{bc} = \bar{p}_\infty + \frac{p_{bc} - p_\infty}{a_0^2}
\]

\[
(p\tilde{u})_{bc} = \bar{p}_{bc} \left( \tilde{u}_\infty - \hat{n} \frac{S_{ax}}{S_s} \left( \bar{p}_\infty - \bar{p}_{bc} \right) \right)
\]

\[
(p\tilde{v})_{bc} = \bar{p}_{bc} \left( \tilde{v}_\infty - \hat{n} \frac{S_{ay}}{S_s} \left( \bar{p}_\infty - \bar{p}_{bc} \right) \right)
\]

\[
(p\tilde{w})_{bc} = \bar{p}_{bc} \left( \tilde{w}_\infty - \hat{n} \frac{S_{az}}{S_s} \left( \bar{p}_\infty - \bar{p}_{bc} \right) \right)
\]

\[
e_{bc} = \bar{p}_{bc} \left( \frac{\bar{p}_{bc}}{(\gamma - 1)\bar{p}_{bc}} + \frac{1}{2} (\tilde{u}_{bc}^2 + \tilde{v}_{bc}^2 + \tilde{w}_{bc}^2) \right)
\]

\[
\tilde{T}_{bc} = \frac{\bar{p}_{bc} \tilde{M}_\infty^2}{\bar{p}_{bc} R}
\]
\[ \tilde{a}_{bc} = \left( \frac{\bar{\rho}_{bc}}{\tilde{\rho}_{bc}} \right)^{\gamma/2} \]

where the subscript \( bc \) indicates values at the boundary and the subscript \( int \) indicates values at the first interior point.

For subsonic outflow, the primitives are determined as follows,

\[ \bar{p}_{bc} = \bar{p}_{\infty} \]

\[ \tilde{\rho}_{bc} = \bar{p}_{\infty} + \frac{p_{bc} - p_{\infty}}{a_0^2} \]

\[ (\bar{\rho} \bar{u})_{bc} = \bar{p}_{bc} \left( \bar{u}_{int} - \hat{n} \left( \frac{\bar{p}_{int} - \bar{p}_{bc}}{S_a} \right) \right) \]

\[ (\bar{\rho} \bar{v})_{bc} = \bar{p}_{bc} \left( \bar{v}_{int} - \hat{n} \left( \frac{\bar{p}_{int} - \bar{p}_{bc}}{S_a} \right) \right) \]

\[ (\bar{\rho} \bar{w})_{bc} = \bar{p}_{bc} \left( \bar{w}_{int} - \hat{n} \left( \frac{\bar{p}_{int} - \bar{p}_{bc}}{S_a} \right) \right) \]

\[ e_{bc} = \bar{p}_{bc} \left( \frac{1}{\gamma - 1} \right) + \frac{1}{2} \left( \bar{a}_{bc}^2 + \bar{v}_{bc}^2 + \bar{w}_{bc}^2 \right) \]

\[ \tilde{T}_{bc} = \frac{\bar{p}_{bc} \tilde{M}_{\infty}^2}{\tilde{\rho}_{bc} R} \]

\[ \tilde{\rho}_{bc} = \left( \frac{\bar{\rho}_{bc}}{\tilde{\rho}_{bc}} \right)^{\gamma/2} \]

The slip boundary is based on the one-dimensional approximation of the inviscid equations and specifies the primitive variables as follows,
\[
\bar{p}_{bc} = \bar{p}_{\text{int}} + \frac{\bar{p}_{bc} - \bar{p}_{\text{int}}}{\bar{a}_{\text{ref}}^2} \quad (1)
\]

\[
\bar{e}_{bc} = \frac{\bar{p}_{bc}}{(\gamma - 1) \bar{p}_{bc}} \quad (2)
\]

\[
(\bar{p}\bar{u})_{bc} = \bar{p}_{bc} \left( \bar{u}_{\text{int}} - \bar{V}_a \frac{S_{ax}}{S_a} \right) \quad (3)
\]

\[
(\bar{p}\bar{v})_{bc} = \bar{p}_{bc} \left( \bar{v}_{\text{int}} - \bar{V}_a \frac{S_{ay}}{S_a} \right) \quad (4)
\]

\[
(\bar{p}\bar{w})_{bc} = \bar{p}_{bc} \left( \bar{w}_{\text{int}} - \bar{V}_a \frac{S_{az}}{S_a} \right) \quad (5)
\]

where

\[
\bar{p}_{bc} = \bar{p}_{\text{int}} - \hat{n}_{bc} \bar{p}_{\text{ref}} \bar{a}_{\text{ref}} \bar{V}_a \quad (6)
\]

\[
\bar{V}_a = \frac{\bar{V}_{\text{int}} \cdot \hat{S}_a}{|\hat{S}_a|} \quad (7)
\]

and the user-defined freestream conditions are used as reference conditions.

A zeroth-order extrapolation was applied to pressure and temperature for the adiabatic, no-slip wall. For purposes of this study, only adiabatic wall conditions were considered. Smith’s turbulence model modifies the pressure as follows,

\[
\bar{p}_{bc} = \bar{p}_{\text{int}} + \frac{1}{3} \left( (\bar{p}k)_{\text{int}} - (\bar{p}k)_{bc} \right) \quad (8)
\]

The primitive variables \(\rho u, \rho v, \rho w\) are set to zero and the internal energy is handled in the same manner as the slip boundary stated above.

For the symmetry boundary, a zeroth-order extrapolation was applied to density and pressure with the velocities determined as follows,
\[ \text{and finally, the internal energy is} \]

\[ \bar{\varepsilon}_{bc} = \bar{\varepsilon}_{\text{int}} - \frac{1}{2} \rho_{\text{int}} \left| \vec{V}_{\text{int}} \right|^2 + \frac{1}{2} \rho_{bc} \left| \vec{V}_{bc} \right|^2 \]  

\[ (12) \]

For the flat plate with a jet, the only additional boundary condition was the boundary at the inlet of the nozzle where total pressure, \( p_t \), and total temperature, \( T_t \), were specified. Falcon assumes at the linearization or reference point the following characteristic equation is constant:

\[ \frac{\vec{V}_{\text{int}} \cdot \vec{S}_a}{|\vec{S}_a|} + \frac{\bar{p}_{\text{ref}}}{\bar{\rho}_{\text{ref}} \bar{a}_{\text{ref}}} = B \]  

\[ (13) \]

Given

\[ \tilde{T}_t = \bar{T}_t + \frac{\gamma - 1}{2\gamma} \frac{\bar{V}^2}{R} \quad \quad \frac{\bar{p}}{\bar{T}} = \left( \frac{\bar{T}}{\bar{T}_t} \right)^{\gamma \gamma^{-1}} \]

the following equation results,

\[ \frac{2\gamma R}{\gamma - 1} \left( \frac{\bar{p}}{\bar{T}} \right)^{\gamma \gamma^{-1}} + \left( \frac{\bar{p}_t}{\bar{p}} \right)^2 \bar{M}^2 - 2B \bar{M}^2 = 0 \]  

\[ (14) \]
This equation was differentiated and solved for \( p/p_t \). Once \( p \) is known, \( T \) and \( V \) can be found from the previous equations and if the user inputs the three components of the velocity vector, these are used to determine a unit vector and \( V \) is used as the magnitude of the velocity vector at the boundary.

4.2 GASP Model

4.2.1 Equations of Motion

The GASP solver was developed and is maintained by Aerosoft, Inc. in Blacksburg, Virginia. GASP is a three-dimensional full Navier-Stokes solver modeling the Reynolds-averaged conservation equations exactly the same as the Falcon code except that a total energy equation instead of a total enthalpy equation is used:

\[
\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_i} (\rho \bar{u}_i) = 0
\]

\[
\frac{\partial}{\partial t} (\rho \bar{u}_i) + \frac{\partial}{\partial x_j} (\rho \bar{u}_i \bar{u}_j + \delta_{ij} \rho) = \frac{\partial}{\partial x_j} \left[ \bar{\varepsilon}_{ij} - \rho \bar{\varepsilon} \right]
\]

\[
\frac{\partial}{\partial t} (\rho \tilde{\varepsilon}_0) + \frac{\partial}{\partial x_j} \left[ \rho \bar{u}_j \tilde{\varepsilon}_0 + \rho \bar{u}_j \bar{\varepsilon}_0 + \rho \bar{u}_j \right] = \frac{\partial}{\partial x_j} \left( \mu_j \bar{\varepsilon}_j \right) - \frac{\partial q_j}{\partial x_j}
\]

\[
\bar{p} = \rho R \tilde{T}
\]

Fluctuations are modeled as follows,

\[
\rho \bar{u}_i \bar{u}_j = \mu_i \left( \frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i} - \frac{2}{3} \delta_{ij} \frac{\partial \bar{u}_k}{\partial x_k} \right) - \frac{2}{3} \delta_{ij} \rho \bar{k}
\]

\[
\tilde{\varepsilon}_0 = \tilde{h}_0 + \frac{\bar{p}}{\bar{\rho}}
\]
These equations are discretized in the same manner as the Falcon code using the MUSCL upwind extrapolation, Roe’s flux splitting, forward difference operator for the time derivative and a second-order central difference operator for the viscous terms. An implicit solver is used to arrive at a steady-state solution in the same manner as Falcon.

4.2.2 Wilcox’s k-ω Turbulence Model

The user can choose from several turbulence models in GASP. The Wilcox (1998) k-ω two-equation turbulence model determines the turbulent viscosity from the following equations used to solve the turbulent kinetic energy $k$ and the specific dissipation rate $\omega$

\[
\frac{\partial k}{\partial t} + \vec{u}_j \frac{\partial k}{\partial x_j} = \tau_{ij} \frac{\partial \vec{u}_i}{\partial x_j} - \beta^* k \omega + \frac{\partial}{\partial x_j} \left[ (\nu + \sigma^* \nu_i) \frac{\partial k}{\partial x_j} \right]
\]

\[
\frac{\partial \omega}{\partial t} + \vec{u}_j \frac{\partial \omega}{\partial x_j} = \frac{\alpha}{k} \tau_{ij} \frac{\partial \vec{u}_i}{\partial x_j} - \beta \omega^2 + \frac{\partial}{\partial x_j} \left[ (\nu + \sigma \nu_i) \frac{\partial \omega}{\partial x_j} \right]
\]
The turbulent viscosity is related to the turbulent kinetic energy and the turbulent frequency by

$$\nu_t = \frac{k}{\omega}$$

The closure coefficients and other details of this model can be found in Wilcox (1998).

4.2.3 Menter's $\kappa$-$\omega$ Turbulence Model

Menter’s (1993) $k$-$\omega$ turbulence model uses the following equations to determine $k$ and $\omega$.

$$\frac{\partial(\bar{p}k)}{\partial t} + \bar{u}_j \frac{\partial(\bar{p}k)}{\partial x_j} = \tau_{ij} \frac{\partial \bar{u}_i}{\partial x_j} - \beta^P \bar{p}k\omega + \frac{\partial}{\partial x_j} \left[ \left( \mu + \sigma_\omega \mu_\omega \right) \frac{\partial k}{\partial x_j} \right]$$

$$\frac{\partial(\bar{p}\omega)}{\partial t} + \bar{u}_j \frac{\partial(\bar{p}\omega)}{\partial x_j} = \frac{\gamma}{\nu_t} \tau_{ij} \frac{\partial \bar{u}_i}{\partial x_j} - \beta \bar{p}\omega^2 + \frac{\partial}{\partial x_j} \left[ \left( \mu + \sigma_\omega \mu_\omega \right) \frac{\partial \omega}{\partial x_j} \right] + 2 \bar{p}(1 - F_i)\sigma_\omega^2 \frac{1}{\omega} \frac{\partial k}{\partial x_j} \frac{\partial \omega}{\partial x_j}$$

The turbulent viscosity is related by the same equation as Wilcox’s k-w model.

$$\mu_t = C_\mu \frac{\bar{p}\sqrt{k}}{\omega}$$

The closure coefficients and other details of this model can be found in Menter (1993).

4.2.4 Spalart-Allmaras’ One-Equation Turbulence Model

The Spalart-Allmaras (1994) model is a one-equation turbulence model that is used to determine $\tilde{v}$ as follows:

$$\frac{\partial \tilde{v}}{\partial t} + \bar{u}_j \frac{\partial \tilde{v}u_j}{\partial x_j} = \frac{\partial \tau_{ij}^v}{\partial x_j} + C_{b1} \tilde{v} - C_{w1} f_w \left( \frac{\tilde{V}}{d} \right)^2 + \tilde{v} \left[ \nabla \cdot \tilde{v} \right] + \frac{C_{b2}}{\sigma} \left( \nabla \tilde{v} \right)^2$$

The turbulent viscosity is related to $\tilde{v}$ by
\[
\mu = \varrho \nu f_{vl}
\]

where \( f_{vl} \) is a damping function defined as

\[
f_{vl} = \frac{\chi^3}{\chi^3 + C_{vl}^3}
\]

The term \( \chi \) is a kinematic viscosity ratio defined as

\[
\chi = \frac{\bar{\nu}}{\nu}
\]

The closure coefficients and other details of this model can be found in Spalart and Allmaras (1994).

4.2.5 Boundary and Initial Conditions

The boundary and initial conditions for GASP are the same as for Falcon, shown in Figure 4.1 and detailed in section 4.1.3, with the following exceptions. For the slip boundary condition, a zeroth-order extrapolation is applied to the pressure and the entropy with the density and the velocity defined as follows,

\[
\left( \mathbf{p}_{bc} \right)^{\gamma_p} = \left( \frac{\mathbf{p}_{bc}}{\bar{s}_{bc}} \right)^{\gamma_p}
\]

\[
\tilde{u}_{bc} = \tilde{u}_{int} - \hat{n}_x \tilde{u}_{int}
\]

\[
\tilde{v}_{bc} = \tilde{v}_{int} - \hat{n}_y \tilde{u}_{int}
\]

\[
\tilde{w}_{bc} = \tilde{w}_{int} - \hat{n}_z \tilde{u}_{int}
\]

where

\[
\tilde{u}_{int} = \hat{n}_x \tilde{u}_{int} + \hat{n}_y \tilde{v}_{int} + \hat{n}_z \tilde{w}_{int}
\]
For the no-slip boundary condition, a zeroth-order extrapolation is applied to the density, pressure and internal energy while the velocity components at the wall are set to zero. This is the same for the symmetry boundary condition except that the velocity components are zeroth order extrapolated as well except for one component. This depends on the location of the symmetry plane. For example, at a symmetry plane lying in a constant $x$ location,

\[
\tilde{u}_{bc} = -\tilde{u}_{\text{int}}
\]

\[
\tilde{v}_{bc} = \tilde{v}_{\text{int}}
\]

\[
\tilde{w}_{bc} = \tilde{w}_{\text{int}}
\]

For the total pressure and temperature boundary condition, total pressure and total temperature were specified while the static pressure, density and velocity components are determined as follows,

\[
\tilde{p}_{bc} = \tilde{p}_{\text{int}}
\]

\[
\tilde{T}_{bc} = \frac{\tilde{T}_{\text{bc}}}{\left[\left(\frac{\tilde{p}_{\text{bc}}}{\tilde{p}}\right)_{\text{bc}}\right]^{\frac{\gamma-1}{\gamma}}}
\]

\[
\tilde{\rho}_{bc} = \frac{\tilde{p}_{bc}}{R\tilde{T}_{bc}}
\]

\[
\tilde{M}_{bc}^2 = \frac{2}{\gamma-1} \left[\left(\frac{\tilde{T}_{\text{bc}}}{\tilde{T}}\right)_{\text{bc}} - 1\right]
\]

\[
\tilde{a}_{bc}^2 = \gamma R\tilde{T}_{bc}
\]
\[
\tilde{u}_{bc} = \tilde{M}_{bc} \tilde{u}_{bc} \left( \frac{\tilde{u}_{\infty}}{\tilde{M}_{\infty} \tilde{a}_{\infty}} \right)
\]

\[
\tilde{v}_{bc} = \tilde{M}_{bc} \tilde{v}_{bc} \left( \frac{\tilde{v}_{\infty}}{\tilde{M}_{\infty} \tilde{a}_{\infty}} \right)
\]

\[
\tilde{w}_{bc} = \tilde{M}_{bc} \tilde{w}_{bc} \left( \frac{\tilde{w}_{\infty}}{\tilde{M}_{\infty} \tilde{a}_{\infty}} \right)
\]

4.3 Validation

4.3.1 Laminar Flow Over a Flat Plate

Falcon and GASP were applied to flow over a flat plate to obtain undisturbed laminar boundary layer results as a preliminary step to simulating more complex flow fields. Figures 4.2 and 4.3 show the results of laminar simulations for an incoming flow at Mach 2 and Mach 4, respectively, both at a Reynolds number of 328,000/m with an adiabatic wall condition. In Figures 4.2 and 4.3, the velocity is normalized by the incoming freestream velocity and it is plotted against the non-dimensional wall coordinate, \( \eta \), defined as:

\[
\eta = \frac{y}{x} \sqrt{Re_x}
\]

\[
Re_x = \frac{p_{\infty} \tilde{V}_{\infty} x}{\mu_{\infty}}
\]

The computed profiles were compared with Van Driest’s (1952) analytical results at 0.15 and 0.46 m from the leading edge of the plate. The computed profiles for Mach 2 flow shown in Figure 4.2 compare well with the Van Driest profiles except for the GASP
calculation at 0.46m. All of the computed profiles for Mach 4 flow shown in Figure 4.3 compare well with the Van Driest profile.

The boundary layer thickness, $\delta$, based on the $0.99U_\infty$ criterion, along the plate is shown in Figure 4.4. The distance along the plate has been normalized by a reference
length, $L_{ref}$, of $0.3048m$. The solid lines are for Mach 2 while the dashed lines are for Mach 4. For each Mach number, Falcon data, and GASP data are compared with the Van Driest theory. At the leading edge, Falcon and GASP predict significantly thicker boundary layers than boundary layer theory.

Figure 4.4 Boundary layer thickness

This discrepancy can be explained by recalling a zero axial pressure gradient is assumed for flow over a flat plate. This assumption fails at the leading edge in supersonic flow due to viscous interaction between the outer inviscid freestream and the viscous boundary layer. The viscous boundary layer displaces the outer inviscid flow as sketched in Figure 4.5.
The deflection of the inviscid freestream generates a shock wave at the leading edge followed by an expansion fan. The shock wave-expansion fan produces an axial pressure distribution in the freestream shown in Figure 4.6 that is impressed on the boundary layer.
Figure 4.6 shows the surface pressure along the plate normalized by ambient pressure. The axial distance is normalized by the reference length. The axial pressure distribution affects the boundary layer thickness which feeds back to the displacement of the inviscid freestream and the shock wave-expansion fan structure. This feedback, or interaction, between the inviscid freestream and the boundary layer is typically referred to as pressure, or viscous, interaction. Anderson (1989) derives the relationship between boundary layer growth and the axial pressure distribution over a flat plate for a laminar boundary layer and concludes that,

\[
\frac{p_e}{p_\infty} = 1 + \frac{\gamma(\gamma+1)}{4} M_\infty^2 \left(\frac{d\delta^*}{dx}\right)^2 + \gamma M_\infty^2 \left(\frac{d\delta^*}{dx}\right)^2 \left(\frac{\gamma+1}{4}\right)^2 + \frac{1}{M_\infty^2 \left(\frac{d\delta^*}{dx}\right)^2}
\]

where \(p_e\) is the pressure at the edge of the boundary layer. This pressure is impressed on the boundary layer and consequently the surface. Anderson shows that the surface
pressure is a function of the square of the boundary layer growth and the square of the freestream Mach number. As the boundary layer grows, $d\delta^*/dx$, tends to zero and the surface pressure tends to ambient. However, at the leading edge where the boundary layer growth is a maximum, the surface pressure is greater than ambient indicating a non-zero axial pressure gradient. Therefore, this assumption made in boundary layer theory breaks down and the numerical results depart from theory.

The pressure rise at the leading edge is evidence of the viscous interaction. Figure 4.6 shows a decrease in pressure at the trailing edge. The trailing edge pressure decrease can be explained by recalling the boundary layer generates a wake aft of the trailing edge of the plate. As the wake closes, an expansion fan is formed. The pressure decrease realized from the presence of this expansion propagates upstream through the subsonic portion of the boundary layer creating sub-ambient pressures at the trailing edge of the plate. This interaction between the inviscid freestream and the viscous boundary layer affects the boundary layer thickness shown in Figure 4.4.

It is seen in Figure 4.4 for the Mach 2 simulation that the agreement between Falcon and theory for the boundary layer thickness improves further downstream as the viscous-inviscid interaction weakens while the agreement between GASP and theory improves downstream yet fails to agree with theory at the trailing-edge. For Mach 4, Falcon and GASP overpredict the boundary layer thickness by approximately 25% at the leading edge and approximately 8% at the trailing edge.

Figure 4.7 compares the computed local skin friction coefficient with that obtained by the Van Driest theory for both Mach 2 and 4. The solid lines are for the
Mach 2 simulations and the dashed for Mach 4. The distance along the plate is normalized by the reference length.

![Graph of local skin friction coefficients](image)

**Figure 4.7 Local skin friction coefficients**

The agreement between the Falcon and GASP computations and the integral solution of the boundary layer equations given by Van Driest (1952) show that these solvers can reasonably predict boundary layer characteristics of compressible laminar flow fields.

### 4.3.2 Turbulent Flow Over a Flat Plate

Falcon with the Smith $k$-$kl$ model and GASP with the Wilcox $k$-$\omega$ model were used to simulate supersonic turbulent flow over a flat plate. Although this has been done previously by Neel, et al. (2003) using GASP, it is reproduced here to compare with results from the Falcon code and experimental data from Shutts, et al. (1955) and Mabey,

Results from Mach 2.23 and Reynolds number 25,000,000/m simulations are compared in Figures 4.8 and 4.9 with data collected by Shutts, et al. at two locations along the plate, namely \( x = 0.193 \) m and \( x = 0.802 \) m. Figures 4.8 and 4.9 show velocity profiles in wall coordinates defined as

\[
    u^+ = \sqrt{\frac{2T_e}{C_{fs}(T_w - T_e)}} \sin^{-1}\left(\frac{u}{u_e} \sqrt{1 - \frac{T_e}{T_w}}\right)
\]

\[
y^+ = \frac{p_w y u_e}{R T_w} \frac{T_w + C_z}{C_f T_w^{1.5}} \sqrt{\frac{T_w}{T_e}} \frac{C_{fs}}{T_e^2}
\]

The experimental data are shown with diamond symbols while the GASP results are represented by an x and the Falcon calculations are shown by a -. The dashed lines show the law of the wall for the inner viscous sublayer and the logarithmic overlap layer as described by White (1979). The inner viscous sublayer and logarithmic overlap layer are defined as

\[
u^+ = y^+
\]

\[
u^+ = \frac{1}{\kappa} \ln y^+ + C
\]

where

\[
\kappa = 0.4
\]

\[
C = 5.1
\]
The law of the wall is also shown with dashed lines in Figure 4.10 and 4.11 for Mach 4.5, Reynolds number 28,100,000/m simulations compared with data collected by Mabey, et al. at two locations, namely $x = 0.368$ m and $x = 1.384$ m.

Figure 4.8 Turbulent boundary layer profiles, Mach 2.23, $x=0.193$ m
Figure 4.9 Turbulent boundary layer profiles, Mach 2.23, $x=0.802$ m

Figure 4.10 Turbulent boundary layer profiles, Mach 4.5, $x=0.368$ m
Both GASP and Falcon match experimental data and the law of the wall well at Mach 2.23 for the inner viscous and logarithmic overlap regions. Although there are no experimental data in the viscous sublayer, both models predict very similar profiles in that region. GASP does not agree as well as Falcon with experimental data or the law of the wall at both stations for Mach 4.5 in the logarithmic overlap region.

Further comparisons were made by examination of the skin friction coefficients. The local skin friction for compressible turbulent boundary layers is defined as

$$ C_{fs} = \frac{\tau_{w1}}{q_e} $$

where $q_e$ is the dynamic pressure at the edge of the boundary layer and $C_{fs}$ is transformed to the incompressible plane by the Kármán-Schoenherr relation as described by Hopkins and Inouye (1971):
\[
\frac{1}{C_{f_x}} = 17.08 \left( \log_{10} \overline{Re_\theta} \right)^2 + 25.11 \log_{10} \overline{Re_\theta} + 6.012
\]

where \( C_{f_x} \) and \( \overline{Re_\theta} \) are transformed using the Van Driest transformation

\[
\overline{C_{f_x}} = F_c C_{f_x}
\]

\[
\overline{Re_\theta} = F_\theta \overline{Re_\theta}
\]

\[
F_c = rm / \left( \sin^{-1} \alpha + \sin^{-1} \beta \right)^2
\]

\[
F_\theta = \mu_c / \mu_w
\]

where

\[
\alpha = \left( 2A^2 - B \right) / \sqrt{4A^2 + B^2}
\]

\[
\beta = B / \sqrt{4A^2 + B^2}
\]

\[
A = \sqrt{rm/F}
\]

\[
B = \left( 1 + rm - F \right) / F
\]

\[
F = T_w / T_e
\]

\[
r = 0.9
\]

\[
m = 0.2M_e^2
\]

Further details of this transformation can be found in Van Driest (1956) and a summary of this and other skin friction prediction methods can be found in Hopkins and Inouye (1971).

Hopkins and Inouye recommended the Van Driest II theory and compared it to the Kármán-Schoenherr equation (eq. 3 in Hopkins and Inouye (1971)). Figures 4.12 and
4.13 show the transformed local skin friction coefficient for the test data and the CFD results plotted against the transformed Reynolds number based on the momentum thickness and compared with the Kármán-Schoenherr equation for Mach 2.23 and 4.5.

Figure 4.12 Local skin friction coefficients, Mach 2.23
Figures 4.12 and 4.13 show both codes predict skin friction coefficient fairly well at Mach 2.23, but at Mach 4.5, Falcon predicts slightly higher friction coefficients.

4.3.3 Transverse Jet In Supersonic Flow Over a Flat Plate

The complex flow structure of TJISF requires an evaluation of the Navier-Stokes solvers and their turbulence models against experimental data. The Smith model with Falcon, the Wilcox k-ω model, the Menter k-ω model and the Spalart-Allmaras model with GASP were applied to a TJISF over a flat plate. Computations from these models were compared to experimental data from Dowdy and Newton (1963). Dowdy and Newton collected a significant amount of surface pressure data on jets issuing from flat plates in supersonic flow. All four solvers were applied at the test conditions listed in Table 4.1 until convergence was achieved. The resulting upstream pressure distributions along the centerline of the plate are compared in Fig. 4.14.
Table 4.1 Dowdy and Newton Test Conditions

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mach</td>
<td>2.61</td>
</tr>
<tr>
<td>$p_{jet}$ (psf)</td>
<td>43214.4</td>
</tr>
<tr>
<td>$T_{jet}$ (R)</td>
<td>533.67</td>
</tr>
<tr>
<td>$p_{∞}$ (psf)</td>
<td>140.54</td>
</tr>
<tr>
<td>$T_{∞}$ (R)</td>
<td>564.67</td>
</tr>
<tr>
<td>$d_{jet}$ (in)</td>
<td>0.1</td>
</tr>
<tr>
<td>$L_{plate}$ (in)</td>
<td>18</td>
</tr>
<tr>
<td>$W_{plate}$ (in)</td>
<td>7.5</td>
</tr>
</tbody>
</table>

Figure 4.14 Upstream pressure distribution comparisons

In Figure 4.14, the pressure is normalized by the ambient pressure while the distance along the plate is normalized by the diameter of the jet, $d_{jet}$. The Dowdy and Newton experimental data are shown with diamond symbols. The purpose of the comparisons in Figure 4.14 is to show that the model can predict flow field behavior and pressure distributions reasonably well. To that end, examination of the upstream flow
structure and the pressure distribution features will correlate the flow structure with the behavior of the pressure distribution. Figure 4.15 shows the pressure contours and velocity vectors on the centerplane of flow domain for the Falcon computations. The flow structure between each of the CFD solvers is very similar. The Falcon results are used here for illustrative purposes only.

In Figure 4.15, the flow is from left to right. The jet flows from bottom to top and is located in the right bottom corner. This figure shows the freestream passing through an oblique shock upstream of the jet, labeled separation shock, causing the initial pressure rise seen in Figure 4.14. This oblique shock is part of a λ-shock system originating from the near normal freestream shock labeled bow shock. The sharp pressure rise seen at approximately \( x/d_{jet} = -2 \) corresponds to the upstream stagnation point.
point identified in Figure 4.15. Expansions between the initial pressure rise and the stagnation point and the stagnation point and the jet are seen in Figure 4.14 and are part of the separations shown in Figure 4.15 upstream and downstream of the stagnation points.

The pressure distribution calculated by Falcon with the Smith turbulence model agrees well with experimental data. The other solvers underpredict the upstream separation point at \( x/d_{jet} \sim -10 \), underpredict the initial pressure rise and overpredict the stagnation pressure. This comparison shows that two major upstream flow features, namely the initial pressure rise and the stagnation pressure spike, and that the best agreement with the experimental data comes from the Falcon computations. The downstream pressure distribution is very different from the upstream indicating that the flow field is not symmetric about the jet.

The downstream pressure distributions are presented in Figure 4.16.
Figure 4.16 Downstream pressure distribution comparisons

Figure 4.17 Downstream velocity vectors and pressure contours, $M=2.61$, $z/d_{jet}=0.0$
Figure 4.17 shows pressure contours and velocity vectors downstream of the jet with the jet shown in the bottom left corner. In this figure, the pressure contours show the overexpansion of the highly underexpanded jet (PR=307.5). This expansion can be seen in Figure 4.16 between $x/d_{jet} = 0.0$. The overexpansion is followed by a gradual compression until it overshoots ambient pressure and has to expand back to ambient. All of the models overpredict the slope of the recompression and the overshoot except for Falcon. Falcon overpredicts the slope of the recompression, but underpredicts the overshoot. In Figures 4.14 through 4.17, it is seen that all of the solvers can provide reasonable results for transverse jets in a supersonic freestream. Moreover, these figures also show Falcon with the Smith model captures the pressure distributions and flow features relatively better than the other solvers.

4.3.4 Grid Density

The accuracy of the steady-state solution from any CFD solver is directly affected by the density of the grid. As the solver reaches steady state, the accuracy of the solution is dictated by the accuracy of the information passed between cells from iteration to iteration. The accuracy of the flow field information passing from one cell to another is directly affected by grid density. In this study, a relatively coarse grid and a relatively fine grid were exercised with Falcon to ensure grid density did not affect the computational results. A 403 x 45 x 133 grid was used for the flow over the plate while a 17 x 31 x 7 grid was used for the nozzle in the TJISF Falcon simulations. From the steady-state solution of the Mach 2 Falcon TJISF simulation, Figure 4.18 shows velocity vectors and pressure contours in the area of the barrel shock.
The barrel shock is smeared by the coarseness of the grid in the direction normal to the plate. The distribution in the axial direction appears to satisfactorily resolve the flow structure without smearing. Points were added in the normal direction to create a 403 x 63 x 133 grid. This grid was applied the same flow conditions and the solution is shown in Figure 4.19.
A comparison of Figures 4.18 and 4.19 shows that the finer grid satisfactorily resolved the barrel shock and Mach disk. Although this is evidence that a finer grid will resolve shocks, it is more important to properly predict the pressure distribution so that the forces and amplification coefficients are accurate. The pressure distribution at three transverse locations for both grid densities are compared in Figure 4.20. These distributions were integrated and the force and amplification coefficients are given in Table 4.20. The steady-state solutions from the fine and coarse grids are within 2% of each other. The excellent agreement indicates that the range of grid densities has an insignificant effect on the pressures and forces computed by Falcon.

![Figure 4.20 Centerline pressure distributions, fine and coarse grids](image)

Table 4.2 Force and Amplification Coefficients for Grid Density.

<table>
<thead>
<tr>
<th>Grid</th>
<th>$C_N$</th>
<th>$C_{mji}$</th>
<th>$C_T$</th>
<th>$\varepsilon_N$</th>
<th>$\varepsilon_{m}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coarse</td>
<td>-0.00847</td>
<td>.00285</td>
<td>.0184</td>
<td>1.461</td>
<td>1.063</td>
</tr>
<tr>
<td>Fine</td>
<td>-0.00817</td>
<td>.00297</td>
<td>.0184</td>
<td>1.461</td>
<td>1.029</td>
</tr>
</tbody>
</table>
4.3.5 Convergence Criterion

The CFD solvers difference the unsteady Navier-Stokes equations,

\[
\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_i} \left( \rho \bar{u}_i \right) = 0
\]

\[
\frac{\partial \left( \rho \bar{u}_i \right)}{\partial t} + \frac{\partial}{\partial x_j} \left( \rho \bar{u}_j \bar{u}_i + \delta_{ij} \bar{p} \right) = \frac{\partial}{\partial x_j} \left[ \bar{r}_{ij} - \rho \bar{u}_j \bar{u}_i \right]
\]

\[
\frac{\partial}{\partial t} \left( \bar{p} \bar{H} + \frac{\rho \bar{u}_j \bar{u}_i^*}{2} - \bar{p} \right) + \frac{\partial}{\partial x_j} \left[ \bar{p} \bar{u}_j \bar{H} + \bar{u}_j \frac{\rho \bar{u}_j \bar{u}_i^*}{2} + \rho \bar{u}_j \bar{H}^* + q_{ij} - \bar{u}_i \left( \bar{r}_{ij} - \rho \bar{u}_j \bar{u}_i^* \right) \right] =
\]

which are a mixed set of parabolic and hyperbolic partial differential equations (PDEs).

The finite difference approximations of these equations are marched in time from an initial condition to a steady-state solution. Ideally, the time-dependent terms in the differenced N-S would be exactly zero when the steady-state solution is reached. However, in general, a solution can be deemed steady when the time dependent terms are considered negligible compared to the other terms in the equations. Falcon computes the magnitude of the time-dependent terms by taking the difference of the RHS of the N-S equations from iteration n to n+1 for each grid point. Summing the difference for each cell and dividing by the number of grid cells gives an estimate of the magnitude of the time-dependent terms relative to the other terms in the equations. The result of this summation is referred to as the residual. The residual is typically normalized by the difference of the RHS between the initial condition and the first iteration. This is one of the criteria used in this study to determine if a solution has reached steady-state was the
normalized residual. The other criteria were the integrated surface force computed by Falcon at each iteration. If the residual dropped 3 orders of magnitude and the surface forces varied less than 5% over 1000 iterations, the solution was considered steady-state. The normalized residual and integrated surface force histories for the Mach 2.0 flat plate simulation are shown in Figures 4.21 and 4.22 as an example of the criterion used to determine a steady-state solution.

Figure 4.21 Residual history Mach 2.0, PR=2000
Figure 4.21 shows the normalized residual plotted against iteration on a semi-log plot. The normalized residual decreases rapidly in the first 100 iterations, then in a more logarithmic manner thereafter. Figure 4.22 shows the components of the integrated surface forces vary randomly with iteration, settling to steady-state values. When the convergence criteria for both the normalized residual and the integrated surface forces were met, there was a reasonably confident expectation the solution was steady-state and the simulation was stopped. Convergence histories for each simulation are found in Appendix A and B.
CHAPTER 5
RESULTS AND DISCUSSION

In this chapter, the results of the CFD calculations listed in Table 3.2 are discussed in two sections. The first section is made of two subsections: transverse jets issuing from a flat plate into a subsonic freestream and transverse jets issuing from a flat plate into a supersonic freestream. The second section discusses results of a transverse jet issuing from a body of revolution into a supersonic freestream.

5.1 Transverse Jet From A Flat Plate

5.1.1 Subsonic Freestream

In this section, transverse jets issuing into a subsonic freestream are investigated. Calculations with laminar and turbulent boundary layers and perfectly expanded and underexpanded jets are discussed. Previous studies of transverse jets in a subsonic freestream are relevant to the study of TJISF because the counter-rotating jet vortices, a major feature of the flow structure in TJISF, are attributed to studies of transverse jets in subsonic freestreams. Furthermore, this section provides good background to contrast against TJISF.

5.1.1.1 Laminar Boundary Layer Approaching a Jet PR 1.5

A square flat plate 457.2 mm long has a jet orifice located on the centerline and 177.8 mm from the leading edge. The coordinate system used in this study has its origin at the center of the jet exit. The x axis, or axial direction, was directed along the
centerline of the plate toward the trailing edge in the direction of the flow and the \( y \) axis, or transverse direction, was directed along the jet centerline away from the surface in the direction of the jet flow as shown in Figure 5.1. The jet issued from the convergent nozzle shown in Figure 5.2 with dimensions in millimeters.

![Figure 5.1 Coordinate system for flat plate analyses](image)

Figure 5.1 Coordinate system for flat plate analyses
The total pressure at the inlet of the nozzle was 4917.3 Pa (102.7 psf) and the ambient pressure was 3279.8 Pa (68.5 psf). Streamlines through the nozzle for a laminar incoming flow at $M=0.3$ and $PR=1.5$ are shown in Figure 5.3.
The calculations show the streamlines contract through the nozzle and then turn downstream after exiting. Interaction with the freestream, which can be globally described in two parts: the freestream deflecting the jet downstream and the jet obstructing the freestream. Figure 5.3 shows the freestream deflecting the jet. The obstruction of the freestream by the jet however required more examination because it has two components: the obstruction of the inviscid freestream and the obstruction of the approaching boundary layer. To differentiate between these two aspects, the edge of the boundary layer was found using what is defined here as the slope rule. The slope rule states that viscous effects are contained within a layer of flow between the surface and a point in the velocity profile where the slope of the velocity profile, $\frac{\partial u}{\partial y}$, is near zero. For this study, a slope of 0.05 was chosen to define the edge of the boundary layer. Outside of this layer, viscous effects were considered negligible. Using the slope rule, the thickness of the approaching boundary layer was calculated and plotted in Figure 5.4. Such a definition to locate the boundary layer edge which the author finds necessary but which had not been formulated by previous investigators. This criterion is consistent and easy to apply.
Figure 5.4 Boundary layer thickness, laminar, $M=0.3$, PR=1.5

Figure 5.4 plots the boundary layer thickness normalized by the jet diameter against the normalized axial distance from the jet for various normalized lateral distances with the dashed vertical line indicating the upstream location of edge of the jet exit. Figure 5.4 shows that the boundary layer thickness remained fairly constant at approximately 1.3 jet diameters in both axial and lateral directions as it approached the jet until approximately 0.7 diameters upstream of the edge of the jet exit. At this point, the boundary layer thickened rapidly near the centerplane. At lateral distances of 0.5 diameters and greater, boundary layer thickening was delayed and outside of one jet diameter, the boundary layer became thinner. The thickening of the boundary layer was the result of the adverse pressure gradient impressed on the boundary layer by the obstruction of the inviscid freestream by the jet. As the jet exited, it deflected the freestream, as shown in Figure 5.5.
Figure 5.5 Freestream streamlines around jet, laminar

The streamlines in Figure 5.5 were located at 4.2 mm \((y/d_{jet}=1.65)\) from the surface, just outside the boundary layer and, at 25.4 mm \((x/d_{jet}=10)\) upstream of the jet for lateral positions out to 13.4 mm \((z/d_{jet}=5.28)\). These streamlines show an undisturbed pattern up to the point where the jet initially deflected the freestream that was followed quickly by a three dimensional spreading of the streamlines. The initial deflection created a radius of curvature in the streamlines toward the surface and according to Bernoulli’s equation normal to a streamline, see Panton (1984),

\[
\frac{\partial}{\partial n} \left( \frac{p}{\rho} \right) = \frac{U_{\infty}^2}{R},
\]

pressures greater than ambient occurred in front of the jet which created an adverse axial pressure gradient for the approaching boundary layer as well as a favorable lateral pressure gradient.
Figure 5.6 Axial surface pressure, laminar, $M=0.3$, PR=1.5

Figure 5.6 shows normalized surface pressures plotted against the normalized axial distance upstream of the jet at various lateral distances. This adverse gradient thickened the boundary layer by reducing the flow velocities near the edge of the boundary layer, creating a new region of shear stresses. This thickening created reverse flow near the surface as shown by the velocity profiles in Figure 5.7.
Figure 5.7 shows the normalized axial velocity component plotted against the normalized transverse distance on the centerplane, $z/d_{jet}=0.0$, at the onset of flow reversal, $x/d_{jet}=-2.19$, and at a point downstream, $x/d_{jet}=-0.75$. At the beginning of flow reversal, the slope of the velocity profile at the surface was zero. The slope of the downstream profile was negative at the surface with reduced velocities in the thickened region of the profile. Furthermore, as the boundary layer thickened, the transverse pressure became non-uniform. After the flow reversal occurred, pressures elevated in the thickened region of the profile yielding a non-zero transverse pressure gradient in the boundary layer as shown in Figure 5.8.
Figure 5.8 shows the normalized transverse pressure profiles at four lateral locations from the centerplane with the boundary layer thickness and transverse pressure profile upstream of flow reversal plotted for comparison. These profiles show the pressure in the boundary layer had deviated enough (more than 2.5%) from the profile upstream to be considered as a non-zero gradient. The lateral pressure gradient was also affected by the deflection of the freestream as shown in Figure 5.9.
Figure 5.9 shows normalized lateral pressures in the freestream at 4.2 mm ($y/d_{jet}=1.65$) off the surface at two locations: 0.13 mm ($x/d_{jet}=-0.05$) in front of the jet centerline where the initial deflection took place and 4.4 mm ($x/d_{jet}=1.73$) downstream of the jet centerline where the streamline curvature was away from the surface. This figure shows that the three-dimensional curvatures of the freestream streamlines (see Figure 5.5) resulted in a favorable lateral gradient away from the jet at the initial deflection and a favorable lateral gradient toward the jet where the streamlines spread. These lateral pressure distributions resulting from the inviscid freestream/jet interaction play a key role in the interaction between the approaching boundary layer and the jet. The lateral gradients shown here and the axial pressure gradient shown in Figure 5.6 drive the character of the boundary layer in the near field upstream of the jet as shown in Figures 5.7 through 5.8.
To better understand the character of the boundary layer under these conditions, consider a boundary layer on a flat plate in uniform flow. In this case, the boundary layer velocity profile varies in the transverse direction with similar shapes at every axial location. Similarly-shaped two-dimensional profiles can be transformed into a single parameter profile (the similarity parameter) as described by White (1979). If the boundary layer were exposed to an adverse pressure gradient, the boundary layer thickens and the velocity profile becomes dissimilar which eliminates any obvious simplification and leaves truly two dimensional profiles. Furthermore, if a lateral pressure gradient were applied, the boundary layer develops non-uniform crossflow velocities. Crossflow velocities described by Lu (1988) are flow velocities moving lateral to the freestream direction within the boundary layer as a result of a lateral pressure gradient. Non-uniform crossflow velocities develop within the boundary layer because a lateral pressure gradient deflects the lower momentum fluid in the boundary layer laterally more than the fluid at the edge of the boundary layer resulting in non-uniform deflection of the flow through the boundary layer. This non-uniform deflection within a two-dimensional boundary layer gave rise to a complex three-dimensional boundary layer velocity profile near the jet. The three-dimensional character of the boundary layer in this case is illustrated in Figures 5.10 and 5.11.
Figure 5.10 Velocity vectors and lateral vorticity contours on centerplane, laminar

In Figure 5.10, velocity vectors on the centerplane of the flow domain \((z/d_{jet}=0.0)\) are plotted with lateral vorticity contours where the freestream moves from left to right and the jet moves from bottom to top. The freestream vorticity is zero and the vorticity contours shows a rapid rise in vorticity ahead of the jet while the velocity vectors show flow reversal at the surface suggesting the boundary layer may have separated.
Figure 5.11 shows velocity vector and isobars for a horizontal plane 0.27 mm, $(y/d_{jet}=0.11)$, above the surface of the flat plate with flow from left to right and the jet out of the page. The pressure contours show a slight rise in pressure in front of the jet caused by the deflection of the freestream by the jet and the vectors show the crossflow velocities mentioned previously indicating the boundary layer also varies laterally. Figures 5.10 and 5.11 show that the interaction between the jet and the boundary layer resulted in a three-dimensional boundary layer. Determination of three-dimensional separation takes a different approach than two-dimensional separation as described by Lighthill (1963), Tobak and Peake (1982), Legendre (1956) and Davey (1961). Lighthill (1963) and Tobak and Peake (1982) defined three-dimensional separation by the convergence of skin friction lines which, for this case, is presented in Figure 5.12.
Three-dimensional separation is described through a topology of saddle points, nodes, foci, streamlines and separation lines. A line of separation is a particular skin friction line on which other skin friction lines converge. If the skin friction line on which others converge emanates from a saddle point, it is said to be a global line of separation. Otherwise, it is a local line of separation. The convergence of the skin friction lines is the necessary condition for separation of a three-dimensional boundary layer. Furthermore, skin friction topologies with three-dimensional boundary layer separation adhere to certain topological rules (see Tobak and Peake (1982)). However, topological rules for a body in the presence of two streams are not well understood so the approach taken here was to identify the singularities on the surface, then identify the body type simulated by the interacting streams according to the topological rules for three-dimensional separation. Figure 5.12 shows a single saddle point upstream of the jet with a half node
of attachment at the plate/jet junction and two half foci of separation and a half saddle point downstream of the jet at the plate/jet junction. Away from the plate/jet junction, downstream of the jet, there were two more saddle points and one node of attachment for a total of three and a half saddle points and two and a half nodes and foci. In addition to the saddle points, nodes and foci, two pairs of global separation lines, one around the jet and one downstream of the jet, and one pair of local separation lines around the jet are shown in Figure 5.12. These topological features define the phase portrait of this flow field and show that three-dimensional separation occurred both around and downstream of the jet and classify this flow field as a two-dimensional plane cutting a three-dimensional body. This was not surprising as it is well known that the flow field patterns of transverse jets in crossflow and a finite cylinder attached to a surface in crossflow have similar streamline patterns.

The necessary condition for three-dimensional separation to occur is the convergence of skin friction lines (Tobak and Peake 1982). In Figure 5.12, the convergence of the skin friction lines onto the skin friction line emanating from the upstream saddle point satisfied this condition. The skin friction line emanating from a saddle point is a global line of separation. In three-dimensional separation, there are two types of separations, closed and open. A separation zone is considered closed when streamlines from one side of the separation line cannot cross over to the other side of the separation line. Since a global separation line is the base for a stream surface, streamlines cannot cross over and the flow on one side of the global separation line is considered separate from the flow on the other. A separation zone is considered open
when streamlines can cross over. This is possible when a local separation line is present. A local separation line emanates from a singularity other than a saddle point (an attachment node in this case). Figure 5.12 shows a pair of global separation lines around the jet and another pair emanating from the downstream saddle points creating a second “closed” separation zone. Furthermore, the figure shows a pair of local separation lines around the jet.

Global lines of separation also form the base of a stream surface called a dividing surface. The presence of a dividing surface is characteristic of three-dimensional boundary layer separation and can lead to a vortex (Tobak and Peake 1982). For example, as the dividing surfaces emanate from the upstream global separation lines, they can wrap around a vortical core originating from an upstream separation focus into a pair of vortices popularly called horseshoe vortices by Baker (1979). Figure 5.13 shows the horseshoe vortices as they convected downstream in the present subsonic laminar J1 study.
Figure 5.13 Horseshoe and horn vortices, laminar, $M=0.3$

Figure 5.13 shows streamlines wrapping around the dividing surface into horseshoe vortices upstream of the jet, with skin friction lines in black and surface pressure contours in white. The combination of the upstream saddle point and half node of attachment provided the topology required to generate a set of horseshoe vortices with a left running vortex rotating in a *clockwise* direction as shown in Figure 5.13. However, the global separation lines on either side of the jet also wound into the separation foci in a counterclockwise rotation (see Figure 5.12), generating another pair of vortices called horn vortices (Tobak and Peake, 1982) which competed with the horseshoe vortices since the attachment node and separation foci shared a common saddle point.

The dividing surface of a horn vortex always projects away from the surface as it convects downstream forming a horn shape as shown in Figure 5.14.
Figure 5.14 Horn vortices, laminar, $M=0.3$

Figure 5.14 shows the horn-shaped vortices generated by the combination of the upstream saddle point and separation foci which competed for the direction of rotation of the dividing surface with the horseshoe vortices generated by the combination of the same upstream saddle point and the half attachment node. As the horn vortices leave the surface, the left running vortex rotated *counterclockwise* with sufficient strength to unwrap the horseshoe vortices as they approached the separation foci (see red streamline in Figure 5.13). The result of this competition was a dividing surface that rotated clockwise upstream of the jet, but by the time it reached the downstream side of the jet it was rotating *counterclockwise*, a testament to the relative strength of the horn vortices. Downstream of this competition, another pair of dividing surfaces formed from the global separation lines downstream of the jet (see Figure 5.12).
As with the upstream saddle point, both downstream saddle points wound into the separation foci, becoming a part of the horn vortex dividing surface. However, the downstream saddle points also acted in combination with the downstream attachment node to generate two pairs of vortices downstream of the jet. Similar to the upstream pair, the line between the left saddle point and the attachment node split the dividing surface into a *clockwise* rotating vortex convecting *downstream* and a *clockwise* vortex convecting *upstream*. The same occurred on the right side of the symmetry plane with the vortices rotating *counterclockwise* therefore there were two pairs of vortices were generated. Since no reference has been made of two pairs of counter-rotating vortices moving in opposing directions downstream of the jet by Fric and Roshko (1989), Moussa, et al (1977), McMahon, et al (1971) or Pratte and Baines (1967), the vortices convecting upstream toward the jet will be referred to as the near field wake vortices and the vortices convecting downstream away from the jet will be called the far field wake vortices.

Streamlines illustrating the near field wake vortices are shown in Figure 5.15.
Figure 5.15 Near field wake streamlines, laminar, $M=0.3$

These streamlines describe the dividing surface wrapping up and convecting upstream toward the jet. These vortices continue upstream toward the half saddle point where they wound into the horn vortices (not shown).
Figure 5.16 Far field wake streamlines, laminar, $M=0.3$

Figure 5.16 shows streamlines downstream of the jet in the far field. Similar to the near-field wake vortices, this pair of vortices was symmetric about the centerplane but convected downstream to the trailing edge of the plate. Although the far-field wake vortices are flow structure associated with jet interaction (JI), in this case, they had little interaction with the jet in the near field and therefore little effect on the obstruction component of jet interaction forces. Later, under different flow conditions, it will be shown that these vortices do impact JI forces, but for that case, the far-field wake vortices only impact downstream bodies such as protuberances or aerodynamic control surfaces, a subject beyond the scope of this study. Since the JI obstruction component of is the primary focus of the present study, discrimination between the near and far field regions was needed. The three saddle points with some guidance from the lines of separation were used to define the “near” field as shown in Figure 5.17.
Figure 5.17 Near field definition, laminar, $M=0.3$

Figure 5.17 shows the skin friction lines in black, the surface pressure contours in white and the near field boundary in bold red. The impact of the flow field outside of this boundary was considered insignificant to JI forces; however, a strong connection was indicated between the three dimensional separation zone, the perturbation of the surface pressure and the obstruction component of the jet interaction force since a majority of the separation zones and surface pressure perturbations were within this boundary. Because of this connection, an effort to examine the flow within the separation zone was made to better understand the details that gave rise to the obstruction component of JI. Velocity profiles in the separated zone around the jet are plotted in Figure 5.18.
It was postulated that the component of the streamwise velocity parallel to the surface in the boundary layer around the jet could be used to identify the edge of the boundary layer and identify flow characteristics within the separation zone. The parallel component of the streamwise velocity, defined as

$$U_y = \sqrt{u^2 + w^2}$$

is plotted in Figure 5.18 at various distances downstream of the half attachment node. Figure 5.18 shows the parallel component of the normalized streamwise velocity plotted against the normalized transverse direction at a lateral distance of 0.1 diameters from the edge of the jet. The edge of the viscous layer was too ambiguous to define with the slope rule as there were three, sometimes five, possible choices for the edge in these profiles.
If the furthest point were chosen as the edge in order to encompass all significant shear stresses in the viscous layer, the viscous layer could not be characterized as similar, but it would have two characteristics: a thickness greater than the penetration height of the jet as shown in Figure 5.19 and a three dimensional pressure gradient as shown in Figure 5.20.

Figure 5.19 Jet penetration-boundary layer thickness comparison, laminar, $M=0.3$

Figure 5.19 also shows the trajectory of the center streamline of the jet compared to the boundary layer thickness determined using the slope rule when the layer is defined to contain all significant shear stresses. This definition of the boundary layer includes a significant portion of the freestream and the whole of the jet. Clearly, proper application of the boundary layer equations is not possible.
Figure 5.20 Pressure distribution in near field, laminar, \( x/d_{jet} = 0.0 \)

Figure 5.20 shows the transverse pressure distribution on the right axis and the lateral surface pressure distribution on the left axis. This figure shows that the pressure distribution in the near field varies 3 or 4 diameters in both the transverse and lateral directions indicating again the edge of the boundary layer defined in this manner cannot satisfy the zero transverse pressure gradient assumption necessary to analyze this flow with conventional boundary layer theory.

Alternatively, if the closest point to the surface were chosen, there would be an admission that the freestream contained significant shear stresses eliminating the application of a simplified, inviscid model of the freestream. Either choice presented a situation where a significant region of near-field flow could not be properly characterized as either boundary layer or inviscid freestream. At this point, it was considered more practical to treat the whole near-field region as a three-dimensional viscous region where streamlines, skin friction lines and surface pressures would be more productive to
describing flow behavior than trying to fit conventional boundary layer theory and/or inviscid flow analysis to it.

![Figure 5.21 Downstream surface pressures, laminar](image)

Figure 5.21 plots surface pressure contours around the jet. The pressure contours darken as the surface pressures decreased and their shapes were reminiscent of pressure contours around a cylinder in crossflow. The pressure decreased from a maximum upstream pressure at the edge of the jet exit to a point of minimum pressure where the horn vortices originated. Downstream of the horn vortices, the surface pressure recovered similar to a cylinder in crossflow, but the downstream separation zone prevented the surface pressure from reaching ambient at the half saddle point downstream of the jet. Along with the surface pressure, the computations also showed that as the horn vortices left the surface, the jet wrapped into them, creating a large pair of large counter-
rotating vortices well documented by Fric and Roshko (1989), Gruber, et al. (1995) and many others researchers too numerous to mention here.

Figure 5.22 Horn and jet vortices, laminar, $M=0.3$

Figure 5.22 shows jet streamlines beginning around the circumference of the jet exit and the same skin friction lines shown in Figure 5.12 together with streamlines from the horn vortices. These streamlines show the jet wrapped around the horn vortices generating the counter-rotating jet vortices. Although the close proximity of the horn and jet vortices ultimately leads to some mixing of the freestream and the jet, the dividing surface of the horn vortex created a barrier between the jet and the horn vortices preventing any significant mixing in the near field. However, examination of the velocity profile in the $y$-plane through the horn/jet vortices downstream of the jet (Figure 5.23) shows a significant region of the vortex did not obey the irrotational vortex velocity
profile, $V = C/r$, indicating that the two streams mixed through diffusion in a shear layer between the vortical core and ideal vortex as the vortices convected downstream.

Figure 5.23 Horn/jet velocity profile, laminar, $M=0.3$, $x/d_{jet}=1.16$
Furthermore, the trajectory of the horn vortices kept the jet sufficiently close to the surface, promoting intimate contact between the jet and the wake vortices. Figure 5.23 shows velocity vectors downstream of the jet exit at \(x/d_{jet}=1.16\) plotted with the streamlines shown in Figure 5.22 and a velocity profile through the horn/jet vortices. The jet streamlines clearly wrap around the horn vortices as the jet turned downstream and the velocity profile shows the vortical core and ideal vortex regions of the vortices. As the vortices proceed downstream, the shear forces diffused the velocity gradients and gas concentrations promoting mixing, but with no significant impact to JI forces.

The flow structures responsible for the modification of the surface pressures in the near field were the three-dimensional separation zone around the jet and the three-dimensional separation zone behind the jet including the horseshoe, horn, near-field and far-field wake vortices as well as the attachment nodes, separation foci and saddle points. The extent of the influence of these separation zones and topological features on the surfaces pressures and JI were quantified here by calculating the amplification coefficients of the jet. The amplification coefficients are defined as

\[
\varepsilon_N = \frac{C_T + C_{Nji}}{C_T} \quad \varepsilon_m = \frac{C_T \frac{I_{noz}}{L_{ref}} + C_{mji \_LE}}{C_T \frac{I_{noz}}{L_{ref}}}.
\]

where

\[C_{Nji} = C_{N\_JetOn} - C_{N\_JetOff} - C_T \]

\[C_{mji} = C_{m\_JetOn} - C_{m\_JetOff} - C_T \frac{I_{noz}}{L_{ref}}.\]
Table 5.1 shows the thrust coefficient, force coefficient and moment coefficient along with the amplification coefficients for a perfectly expanded jet (PR=1.5) issuing into a laminar subsonic (M=0.3) freestream over a flat plate.

Table 5.1 Coefficients For A Laminar Subsonic Freestream.

<table>
<thead>
<tr>
<th>$C_{Nji}$</th>
<th>$C_{mji}$</th>
<th>$C_T$</th>
<th>$\varepsilon_N$</th>
<th>$\varepsilon_m$</th>
</tr>
</thead>
<tbody>
<tr>
<td>+2.02e-4</td>
<td>+1.22e-4</td>
<td>-2.00e-4</td>
<td>-0.01</td>
<td>-0.57</td>
</tr>
</tbody>
</table>

Amplification coefficients less than 1.0 indicate that the JI force opposed the force of the jet (i.e. attenuation) while amplification coefficients greater than 1.0 indicate that the JI force aided the force of the jet (i.e. amplification). Amplification coefficients less than 0.0 indicate the direction of the jet thrust has been overwhelmed by JI forces and the resultant direction opposes the direction of the jet. For the conditions of this simulation, the jet thrust was completely nullified and the moment calculated around the $z$-axis at the leading edge of the plate was reversed by more than 55%. This overwhelming impact on the jet thrust performance shows how important JI forces and moments are to the operation of reaction jet control systems and how critical it is to understand the flow structure in the near field of the jet.

In the following section, a subsonic turbulent boundary layer approaching a perfectly expanded jet is investigated and compared to the flow structure of these calculations to clarify the impact of the state of the boundary layer on perfectly expanded transverse jets in subsonic freestreams.
5.1.1.2 Turbulent Boundary Layer Approaching A Jet PR 1.5

The same flow domain was used in the calculation of a flow at the same freestream Mach and jet pressure ratio, but with a Reynolds number of two million to simulate a turbulent approaching boundary layer and investigate the effect of the state of the boundary layer on the flow structure in the near field. Maintaining the same freestream Mach number of 0.3 with a different Reynolds number required an ambient pressure of 6555.3 Pa (1369.1 psf) and to maintain the same jet pressure ratio (1.5), The total pressure at the inlet of the nozzle increased to 9832.6 Pa (2053.6 psf).

Figure 5.24 Jet streamlines in nozzle, turbulent, $M=0.3$, PR=1.5

Figure 5.24 shows streamlines through the nozzle for calculations with a turbulent boundary layer approaching the jet. The streamlines deflect in a similar manner to the laminar case (see Figure 5.3) suggesting a generally similar interaction between the jet
and the inviscid freestream. The interaction between the jet and the turbulent freestream boundary layer is shown by plotting the boundary layer thickness in Figure 5.25.

Figure 5.25 Boundary layer thickness, turbulent, M=0.3, PR=1.5

Figure 5.25 shows normalized boundary layer thickness plotted against the normalized axial distance at various lateral distances with the laminar boundary layer thickness at the centerplane (z/d_{jet}=0.0) plotted for comparison. The turbulent boundary layer is approximately 20% thinner than the laminar boundary layer, but both thicken at very nearly the same axial location and the adverse pressure gradient in the turbulent calculations followed a similar trend to the laminar, with slightly higher magnitudes as shown in Figure 5.26.
Figure 5.26 Axial surface pressure, turbulent, $M=0.3$, PR=1.5

Figure 5.26 shows normalized surface pressure plotted against the normalized axial distance at various lateral distances with the centerplane laminar profile ($z/d_{jet}=0.0$) plotted for comparison. This figure shows that the adverse pressure gradient weakened at distances lateral to the jet and the comparison with the laminar calculations shows the turbulent boundary layer faced a more adverse pressure gradient. The stronger gradient was a result of the absence of flow reversal upstream of the jet as shown in the velocity profiles of Figure 5.27.
Figure 5.27 shows the normalized axial velocity plotted against the normalized transverse distance at various axial locations along the centerplane ($z/d_{jet}=0.0$). Although the boundary layer thickened as it approached the jet, the velocity gradient at the surface remained positive. These profiles show that no flow reversal had taken place upstream of the jet which strengthened the adverse pressure gradient within the boundary layer. Within the boundary layer, the obstruction of the jet forced the conversion of kinetic energy into enthalpy as it approached the jet which was not possible in the laminar calculations. In the laminar case, the adverse gradient had reversed the direction of a thin layer of flow in the boundary layer precluding any further conversion of kinetic energy to
enthalpy. In the turbulent calculations, with no flow reversal, a stronger adverse gradient was sustained by the more robust flow.

Figure 5.28 shows the normalized pressure plotted against the normalized transverse distance at the leading edge of the jet exit along the centerplane ($z/d_{jet}=0.0$). This figure shows that the turbulent interactions induced pressures that were greater than that in laminar interactions near the surface. The pressure near the surface was greater for the turbulent boundary layer for two reasons: the momentum available in the turbulent boundary layer was greater due to turbulent mixing and second, no flow reversal was present. Higher momentum led to higher pressures as the interaction slowed the flow while the greater streamline deflection contributed a smaller radius of curvature toward...
the surface. On the other side of the interaction, the elevated pressures near the surface in the turbulent case deflected the jet streamlines more than the laminar case. As the jet streamlines saw more deflection in the turbulent case, the freestream streamlines away from the surface deflected less, making their curvatures milder and thus reducing the pressure in the boundary layer away from the surface.

Figure 5.29 shows the freestream streamlines just outside the turbulent and laminar boundary layers with the jet streamlines at the leading edge of the jet exit and a streamline inside each boundary layer. This figure shows that the turbulent boundary layer deflected the jet streamline more than the laminar boundary layer and the curvature of the freestream streamline was milder in the turbulent case than the laminar cases.
leading to a weaker lateral pressure gradient with the turbulent boundary layer as shown in Figure 5.30.

Figure 5.30 shows normalized pressures plotted against normalized lateral distance at two axial locations for both the turbulent and laminar cases. This figure shows that the turbulent boundary layer resulted in slightly weaker lateral pressure gradients both upstream and downstream of the jet. The application of this lateral pressure gradient to the turbulent boundary layer produced crossflow velocities similar to the laminar case.
Figure 5.31 shows velocity vectors and pressure contour calculations for a horizontal plane 0.27 mm ($y/\delta_{jet}=0.11$) above the surface of the plate. The pressure contours were similar to the laminar calculations and the velocity vectors show the crossflow velocities indicating a three-dimensional boundary layer was present in these calculations as well. Skin friction lines calculated around the jet plotted in Figure 5.32 show that the boundary layer separated but the separation zone was not apparently visible.
Two half saddle points, one upstream and one downstream, one saddle point and two half foci of separation, both downstream at the jet/plate junction, were present with a pair of global separation lines emanating from the upstream saddle point and winding into the separation foci. Bifurcations occurred upstream and downstream with the change in boundary-layer state. Upstream, a transcritical bifurcation described by Chapman (1986) changed the half attachment node and saddle point into a half saddle point while downstream, a pitchfork bifurcation (Chapman 1986) changed the two saddle points and the attachment into an single saddle point putting this flow field in the same class as the laminar simulation since there was one more saddle point than nodes and foci. In contrast to the laminar case, there were no attachment nodes upstream or downstream and no global separation lines downstream. In these calculations, the global separation lines around the jet divided the jet and the freestream with no identifiable separation zone.

Figure 5.32 Skin friction lines, turbulent, $M=0.3$, PR=1.5
between them. In addition, no downstream separation lines or separation zones could be identified either. Without separation zones, there were no horseshoe or wake vortices, only horn vortices generated by the dividing surfaces winding into the separation foci. While there was no significant separation around the jet and while the surface pressures were perturbed, the near field region could not be defined by simply following the global lines of separation. The near-field boundary in this case, shown in Figure 5.33, was defined by the half saddle point upstream, the saddle point downstream and a judicious selection of a surface pressure contour to include significant surface pressure perturbations. Although it was not done in this manner for the laminar results, the area defined in the laminar calculation did include nearly all of the pressure contours so it seemed reasonable in this case to use the pressure contours as part of the boundary (in the absence of separation lines).

![Figure 5.33 Near field boundary, turbulent, $M=0.3$, PR=1.5](image)

Figure 5.33 Near field boundary, turbulent, $M=0.3$, PR=1.5
Figure 5.33 shows skin friction lines with the pressure contours in white and the near field boundary in bold red. The near field region defined in this manner had a completely different shape than in the laminar case (see Figure 5.17) and was completely occupied by attached three-dimensional viscous flow instead of a separation zone. However, the boundary layer being separated or attached did not significantly change the character of the velocity or pressure profiles in the near field of the jet. The velocity and pressure profiles in the near field shown in Figures 5.34 and 5.35, respectively, were similar to the laminar case.

Figure 5.34 Velocity profiles, turbulent, PR=1.5, $\Delta z/d_{jet}=0.1$
Figure 5.34 shows the parallel component of the normalized streamwise velocity component defined in the last section plotted against the normalized transverse direction at a lateral distance of 0.1 diameters from the edge of the jet.

Figure 5.35 Pressure distribution around jet, turbulent, PR=1.5, \( x/d_{jet} = 0.0 \)

Figure 5.35 shows the transverse pressure distributions on the right axis and the lateral pressure distribution on the left axis. The velocity profiles in Figure 5.34 show the same dissimilar character as in the laminar case (see Figure 5.18) and the pressure profiles in this figure show the same non-zero pressure gradient seen in the laminar calculation (see Figure 5.20). With the character of these profiles being similar to the laminar calculations, it was no surprise to see the surface pressures in the near field shown in Figure 5.36 similar as well despite the presence of separation zones in the laminar case.
Figure 5.36 shows surface pressure contours around the jet. The pressure contours darken as the surface pressures decrease. These contours are similar to the laminar case (see Figure 5.21) with the exception that the minimum pressure was lower and further downstream. The similarity between these profiles and contours shows the flow in the near field was very similar to the laminar calculations even though the laminar was separated. In the laminar case, there were two separate flow fields around the jet while in turbulent case, there was only one. So regardless of whether the near field had a separation zone or not, the character of the velocity profiles, pressure gradients and surface pressures contours was very much the same.

The state of the approaching boundary layer bifurcated the phase portrait affecting the flow structure around the jet significantly, but the character of the contours and
profiles were similar to the laminar case. The laminar case showed a system of dependent horn-horseshoe vortices with near and far-field wake vortices with four global lines of separation and two local lines of separation while the turbulent calculations showed only horn vortices and two global separation lines, yet the amplification coefficients were very similar.

Table 5.2 shows the thrust coefficient, force coefficient and moment coefficient along with the amplification coefficients, as previous defined, for a perfectly expanded jet (PR=1.5) issuing into laminar and turbulent subsonic (M=0.3) freestreams over a flat plate.

Table 5.2 Coefficients Subsonic Freestreams, PR=1.5.

<table>
<thead>
<tr>
<th>Boundary Layer</th>
<th>$C_{Nji}$</th>
<th>$C_{mji}$</th>
<th>$C_T$</th>
<th>$\varepsilon_N$</th>
<th>$\varepsilon_m$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Turbulent</td>
<td>+2.07e-4</td>
<td>+1.29e-4</td>
<td>-2.15e-4</td>
<td>+0.04</td>
<td>-0.54</td>
</tr>
<tr>
<td>Laminar</td>
<td>+2.02e-4</td>
<td>+1.22e-4</td>
<td>-2.00e-4</td>
<td>-0.01</td>
<td>-0.57</td>
</tr>
</tbody>
</table>

Although the minimum pressure is significantly lower in the turbulent simulation, the lower pressures act over a significantly smaller area than the laminar case which balanced the JI forces and moments between the simulations. In both cases, the amplification coefficients show that the thrust was essentially nullified and the moment about the leading edge of the plate was reversed by the JI force. The turbulent case showed that although no separation zone was present, the downstream surface pressures dominated the near field and, similar to the laminar case, were the root cause of thrust nullification and moment reversal. In the next section, the flow structure and jet interaction for an underexpanded transverse jet issuing into a turbulent subsonic (M=0.3)
freestream are compared to the flow structure of the previous simulations to show the effects of an underexpanded jet on the near field flow structure.

5.1.1.3 Turbulent Boundary Layer Approaching A Jet PR 5

Further investigation of transverse jets in a subsonic freestream was done to examine the effect of jet pressure ratio on JI. Calculations on the same flow domain at the same flow conditions were made with a nozzle inlet total pressure of 327.8 kPa (6845.3 psf) for a jet pressure ratio of 5.0, approximately 3 times greater than the perfectly expanded jet simulation. Figure 5.39 shows streamlines through the nozzle at these conditions.

![Figure 5.37 Jet streamlines in nozzle, turbulent, $M=0.3$, PR=5.0](image)

The increase in jet pressure ratio had two effects: greater jet penetration into the freestream as shown in this figure and the formation of expansion and compression...
waves within the jet plume. Wave formations in the jet are shown in Figure 5.38 by plotting Mach contours on the centerplane of the flow domain.

As the jet exited the nozzle, expansion waves emanated from the edge of the exit reducing the pressure in the jet. These waves traveled across the jet, intersecting the pressure boundary between the freestream and jet, reflecting as compression waves causing the boundary to curve. These compression waves coalesced into an oblique shock and, as shown in Figure 5.38, traveled across the jet intersecting the opposite running oblique shock. As described by Shapiro (1953), the intersecting shocks deflect before reflecting as expansion waves from the pressure boundary. These weaker expansion waves traveled across the jet, intersected the interface boundary (not shown) and reflected as compression waves eventually coalescing into a weaker oblique shock.
This final oblique shock was at a shallow enough angle without forming a Mach reflection and as the pressure within the jet equalized with the freestream, the succession of waves dissipated. Figure 5.39 sketches details associated with the wave formations in the jet flow.

![Diagram of shock train in jet, M=0.3, PR=5](image)

*Figure 5.39 Sketch of shock train in jet, $M=0.3$, PR=5*

The series of shock waves sketched in Figure 5.39 is known as a shock train and the crossing oblique shocks are known as shock diamonds. This figure shows the shocks with dashed lines, the expansion waves as solid lines and flow vectors with arrows and illustrates how increasing the jet pressure ratio completely altered the flow structure within the jet.

The other effect of increasing jet pressure ratio was jet penetration.
Figure 5.40 Jet penetration, turbulent, $M=0.3$

Figure 5.40 shows that the perfectly expanded jet (PR=1.5) penetrated eight diameters into the Mach 0.3 freestream at forty diameters downstream of the jet and the underexpanded jet (PR=5) penetrated almost nineteen diameters, more than 200% further, at the same downstream location. Although the higher jet pressures doubled the jet penetration, the axial pressure gradient weakened.
Figure 5.41 Axial surface pressure, turbulent, $M=0.3$, PR=5

Figure 5.41 shows the normalized surface pressure plotted against the normalized axial distance in front of the jet at various lateral locations, with the surface pressure distributions from the laminar and PR=1.5 calculations at $z/d_{jet}=0.0$ plotted for comparison. The increase in PR resulted in supersonic flow at the exit plane of the nozzle with expansion waves emanating from the exit reducing the pressure within the jet as it expanded into the freestream. The expansion waves caused a favorable pressure gradient for the approaching boundary layer at lateral distances greater than 0.6 diameters. At lateral distances less than that, the jet obstruction caused the surface pressures to rise, creating an adverse pressure gradient before the expansion of the jet was realized. The trend and magnitude are very similar to the laminar calculations until the influence of the jet was realized. However, the boundary layer thickness was not the same as the laminar calculations. The boundary layer thickness five diameters upstream of the jet was very similar to the PR=1.5 calculations as shown in Figure 5.42.
Figure 5.42 Boundary layer thickness, turbulent, \( M=0.3 \), PR=5

The boundary layer was 1 jet diameter thick upstream of the jet, but as it approached the jet, it became thinner at lateral distances greater than 0.6 diameters because of the favorable pressure gradient created by the expansion waves. Near the centerplane, the obstruction of the jet caused a rapid thickening similar to the previous PR=1.5 cases, but the increased PR nearly doubled the thickness. Since the boundary layer thickening was at the same location as the previous cases, examination of skin friction lines shown in Figure 5.43 was done to investigate if any separation had occurred.
Figure 5.43 Skin friction lines around jet, turbulent, $M=0.3$, PR=5

Figure 5.43 shows the skin friction lines on the surface of the plate in the near field of the jet with surface pressure contours plotted in white. The pressure contours are similar to both the laminar and PR=1.5 calculation, but the phase portrait had bifurcated from the PR=1.5 case. Downstream, another pitchfork bifurcation changed the saddle point back to two saddle points and one attachment node, like the laminar topology, and brought back the two global separation lines around the jet, yet no identifiable separation zone between the jet and freestream was present. The total number of saddle points and nodes and foci after this bifurcation were 3 and 2, respectively, keeping this flow field in the same class as the previous calculations. The global separation lines around the jet emanated from the upstream half saddle point, wound into the half separation foci and were the base of a dividing surface coiling into horn vortices.
Figure 5.44 shows streamlines beginning just inside the jet exit and just outside the jet exit. The streamlines just outside the jet exit follow the dividing surfaces emanating from the global separation lines on either side of the jet to form horn vortices, with the jet streamlines rotating into the horn vortices as both pairs of streamlines convect downstream. Downstream of the horn vortices, near-field and far-field wake vortices were present, in contrast to the PR=1.5 calculations. The global separation lines downstream of the jet generated near-field and far-field vortices like those seen in the laminar case (see Figure 5.12).
Figure 5.45 Near field wake vortices, turbulent, $M=0.3$, PR=5

Figure 5.45 shows streamlines following the dividing surfaces of the downstream global lines of separation between the saddle points and the separation foci. In addition, skin fiction lines are shown in black. The streamlines show the vortex pair convected upstream with the left running vortex rotated \textit{clockwise} and the right running vortex rotated counterclockwise. Similar to the near field wake vortices in the laminar calculations, these vortices wound into the horn vortices. The dividing surfaces downstream of the saddle points rotated \textit{clockwise} convecting \textit{downstream} away from the jet as shown in Figure 5.46.
These dividing surfaces coiled up into the far field wake vortices as they convected downstream but, as mentioned previously, these vortices did not affect the near field surface pressures so were of no consequence to the obstruction component of jet interaction. Since far-field wake vortices were of no consequence and no significant separation around the jet was present, the near-field region was defined in a manner similar to the PR=1.5 case, by the saddle points and the judicious selection of a pressure contour. Inclusion of a majority of the pressure contours between the saddle points presented a near field boundary shaped similar to the PR=1.5 calculations.
Figure 5.47 Near field boundary, turbulent, $M=0.3$, PR=5

Figure 5.47 shows the near-field boundary in bold red with skin friction lines in black and surface pressure contours in white. The region outside of this boundary had an insignificant effect on the JI force and moment while within the boundary the surface pressure perturbations dictated the JI force and moment. Furthermore, the two lobe character of the surface pressure contours shown in Figure 5.48 was similar for each simulation indicating the presence or absence of separation had little impact on surface pressures and JI forces and moments in subsonic freestreams. Later, when supersonic freestreams are considered, this will not be the case.
Figure 5.48 Near field surface pressures, turbulent, $M=0.3$, PR=5

Figure 5.48 shows surface pressure contours around the jet. The contours darken as the surface pressures decrease. The minimum pressure around the jet was significantly lower than the PR=1.5 calculations as well as the surface pressure immediately downstream of the jet while the pressure immediately upstream remained nearly the same. Although the pressure around the jet decreased, the same two (2) lobe character seen in the previous calculations was present so it was presumed the character of the velocity and pressure profiles were the similar as well.
Figure 5.49 shows the parallel component of the streamwise velocity normalized by the freestream velocity plotted against the normalized transverse distance. The character of these profiles and the pressure profiles were very similar to the PR=1.5 cases.

Figure 5.49 Velocity profiles, turbulent, PR=5, $\Delta z/d_{jet}=0.1$
Figure 5.50 Pressure distribution around jet, turbulent, PR=5, $x/d_{jet}=0.0$

Figure 5.50 shows normalized pressure plotted against the lateral distance on the left axis and the transverse distance on the right axis. Because of the similarity of these velocity and pressure profiles and contours to the profiles and contours of the previous calculations, it was assumed the amplification coefficients would be similar as well despite the difference in jet pressure ratio and separation topology. Although the difference in jet pressure ratio bifurcated the topology of the skin friction lines, increased jet penetration and generated wake vortices, the surface pressure contours were not altered enough to prevent jet thrust nullification and jet moment reversal. Table 5.3 summarizes the thrust coefficient, force coefficient and moment coefficient along with the amplification coefficients for the three subsonic freestream simulations.
<table>
<thead>
<tr>
<th>Boundary Layer</th>
<th>Pressure Ratio</th>
<th>$C_{Nji}$</th>
<th>$C_{mji}$</th>
<th>$C_T$</th>
<th>$\varepsilon_N$</th>
<th>$\varepsilon_m$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Laminar</td>
<td>1.5</td>
<td>+2.02e-4</td>
<td>+1.21e-4</td>
<td>-2.00e-4</td>
<td>-0.01</td>
<td>-0.55</td>
</tr>
<tr>
<td>Turbulent</td>
<td>1.5</td>
<td>+2.07e-4</td>
<td>+1.29e-4</td>
<td>-2.15e-4</td>
<td>+0.04</td>
<td>-0.54</td>
</tr>
<tr>
<td>Turbulent</td>
<td>5.0</td>
<td>+1.54e-4</td>
<td>+7.79e-4</td>
<td>-1.64e-3</td>
<td>+0.06</td>
<td>-0.22</td>
</tr>
</tbody>
</table>

The state of the boundary layer had little impact on any of the coefficients, but the increase in jet pressure ratio had a significant impact on the force and moment coefficients. The JI force, $C_{Nji}$, decreased 25%, the JI moment, $C_{mji}$, increased 600% while the thrust coefficient, $C_T$, increased an order of magnitude, yet the jet thrust was still nullified and the jet moment still reversed. The force amplification coefficient remained near zero while the moment amplification coefficient was reduced by more than 50%.

For all three simulations examined in this section, the downstream surface pressures which were not significantly affected by the presence or absence of separation or horseshoe or wake vortices dictated the JI force and moment. The fact that the horn and jet vortices were present in all three calculations and the amplification coefficients remained fairly constant over the simulations indicated the horn and jet vortices were the flow structure with the major influence over the downstream surface pressures that drove the amplification coefficients. The conclusion drawn from these simulations was JI has a dramatic effect on the jet force and moment in subsonic freestreams regardless of jet pressure ratio or boundary layer state and the flow structure responsible for the effect was
the horn and jet vortices. In the following sections, JI, surface pressures and flow structure in a supersonic freestream are examined.

5.1.2 Supersonic Freestream

The flow structure in the near field of a transverse jet was very different in supersonic flow. In the following sections, transverse jets issuing into a supersonic freestream are examined. The first subsection compares a transverse jet with a pressure ratio of 5.0 issuing into a Mach 2 freestream with the Mach 0.3, PR=5.0 flow structure. The second subsection sweeps through a range of jet pressure ratio at Mach 2.0 to show the variation in flow structure with jet pressure ratio and the last section sweeps through a range of freestream Mach number at PR 2000 to show variation in flow structure with freestream Mach number.

5.1.2.1 Pressure Ratio 5

To maintain the same Reynolds number and jet pressure ratio at Mach 2, the ambient pressure had to decrease to 9.83 kPa (205.4 psf) and nozzle inlet total pressure had to decrease to 49.16 kPa (1026.8 psf). Streamlines through the nozzle at these conditions are shown in Figure 5.54.
Figure 5.51 Jet streamlines in nozzle, $M=2.0$, PR=5

These streamlines show that jet penetration was dramatically reduced as a result of wave formations in the freestream produced by jet interaction. Figure 5.52 shows velocity vectors and pressure contours on the centerplane of the flow domain upstream of the jet.
The underexpanded jet emerged from the nozzle, in the bottom right corner of Figure 5.52, and obstructed the freestream approaching from the left. The obstruction deflected the supersonic freestream in the transverse and lateral directions generating a three-dimensional shock wave, typically referred to as the bow shock. The bow shock interacted with the approaching boundary layer to create a complex shock/boundary layer interaction region known as a $\lambda$-shock structure. The $\lambda$-shock structure is the result of the interaction between the inviscid freestream and viscous boundary layer. A sketch is made of Figure 5.52 in Figure 5.53 to clarify this interaction and the $\lambda$-shock.
As the bow shock in the inviscid freestream intersected the boundary layer, the boundary layer sensed a rise in pressure altering the velocity profile and thickening the boundary layer as shown in Figure 5.53. As the shock penetrated the boundary layer, it was refracted downstream by the change in Mach number through the boundary layer. The refracted shock turned a portion of the boundary layer flow toward the surface creating a node of attachment (or stagnation point) and an adverse pressure gradient that separated the boundary layer from the surface creating a saddle point in the flow. As will be shown later, the saddle point originated a separation line and a separation zone between the freestream and the jet. Within the separation zone, the node of attachment divided two recirculation regions. The flow to the left of the node turned into an upward recirculation between the saddle point and node. The flow to the right turned...
toward the jet, deflecting immediately upward into the saddle point within the interior of the flow. The flow is then turned back toward the attachment node where the recirculation began. Outside the separation zone, upstream of the saddle point, the thickened boundary layer produced oblique compression waves which coalesced into the bow shock, thereby creating the upstream leg of the $\lambda$. Along with the refracted bow shock, the compression waves completed the $\lambda$-shock structure. This $\lambda$-shock dominated the near field flow structure upstream of the jet and dramatically impacted the jet trajectory.

Figure 5.54 Jet plume Mach contours, turbulent, $M=2.0$, PR=5

Figure 5.54 shows Mach contours on the centerplane of the jet. Compared to the subsonic calculations, the path of the jet was significantly altered by the supersonic freestream and the $\lambda$ shock. The strength of the $\lambda$-shock bent the jet approximately 30°
downstream while the subsonic freestream bent the jet only a few degrees. As the jet was
turned, an oblique shock wave formed within the jet, typically referred to as a barrel
shock, which propagated across the jet as illustrated in Figure 5.55.

Figure 5.55 Sketch of jet and upstream flow structure

As the barrel shock traveled across the jet, expansion waves emanating from the leeward
edge of the jet exit intersected the barrel shock bending it downstream into a Mach disk.
These expansion waves deflected as they passed through the barrel shock generating an
expansion fan in the interaction region which turned the freestream around the jet. As the
jet turned downstream, separation foci (discussed later) near the leeward edge turned the
jet away from the surface generating a second leg of the barrel shock. This second leg
curved away from the surface into the Mach disk completing the incident shock of the
Mach disk. As the reflection from the Mach disk traveled downstream, the left running
reflection weakened by the intersection of the expansion waves and the right running reflection coalescing with compression wavelets emanating from the downstream separation zone (discussed later).

This flow structure just described is considerably different from the underexpanded jet issuing into a subsonic freestream. The presence of wave formations in the freestream produced remarkably different flow characteristics. These wave formations were driven by the fundamental inability of pressure disturbances to propagate upstream in supersonic flow. In no better way can this complex nature of supersonic flow be seen than in Figure 5.56 where skin friction lines are plotted with surface pressure contours.

Figure 5.56 Skin friction lines around jet, $M=2.0$, PR=5
Figure 5.56 shows skin friction lines in black and surface pressure contours in white with the salient features of the separation topology highlighted. This figure shows four saddle points, one node of attachment and two separation foci with two pairs of global lines of separation. The change in the freestream Mach number created three pitchfork bifurcations (Chapman 1986). Downstream, a pitchfork bifurcation changed the two saddle points and attachment node into a single saddle point while another pitchfork bifurcation close to the jet changed the two half separation foci into two separation foci and one saddle point with a third pitchfork bifurcation changing the two half saddle points at the plate/jet junction into two saddle points and one attachment node upstream of the jet classifying this flow field as a two-dimensional plane cutting a three-dimensional body with four saddle points and three nodes and foci. Furthermore, the separation foci moved away from the jet/plate junction and the pair of global separation lines emanating from the new upstream saddle point did not converge to any topological point. Instead, they proceeded downstream separated by nearly seven jet diameters without ever closing on a node or foci. Since this second pair of global separation lines did not converge, a large portion of the plate was covered by the separation zone.

This last bifurcation created a massive separation around the jet spawning horseshoe vortices somewhat similar to those seen in the laminar calculations.
Figure 5.57 Horseshoe vortices, $M=2.0$, PR=5

Figure 5.57 shows streamlines coiling up into horseshoe vortices as they follow the dividing surfaces around the jet into the separation zone and skin friction lines in black and surface pressure contours in white. As in the laminar calculations, the streamlines wrapped under each other producing a *clockwise* left running vortex and a *counterclockwise* right running vortex. However, in contrast to the laminar calculations, this pair of global separation lines was not bound to the separation foci downstream of the jet so the horseshoe vortices proceeded downstream unimpeded by the presence of the horn vortices.

The emergence of the new saddle point created from the $\lambda$ shock structure produced a pair of unbounded global separation lines effectively uncoupling the
horseshoe vortices from the horn vortices allowing the two pairs of vortices to coexist as shown in Figure 5.58.

Figure 5.58 Horseshoe and horn vortices, $M=2.0$ PR=5

Figure 5.58 shows streamlines wrapping around the dividing surfaces emanating from the global separation lines formed between the half saddle point and the two separation foci into horn vortices and streamlines wrapping around the dividing surfaces emanating from the global separation lines originating from the new saddle point into horseshoe vortices. These horseshoe vortices were more pronounced than the horseshoe vortices in the laminar case due to the presence of the $\lambda$ shock. In the laminar case, the saddle point was shared between the attachment node and separation foci. In the present case, there were two upstream saddle points: one acting in combination with the separation foci and one acting in combination with the attachment node. These allowed
the horseshoe vortices to develop as they convected downstream. The $\lambda$ shock altered the flow structure by producing a second upstream saddle point, allowing the horseshoe vortices to develop, forming an upstream attachment node and producing the transverse pressure gradient necessary to deflect the jet. The alteration of the path of the jet relocated the separation foci downstream moving the horn vortices away from the jet/plate junction and significantly reducing the ability of the jet to penetrate the freestream. Figure 5.59 plots jet penetration for the turbulent cases examined thus far.

![Jet penetration, PR=5](image)

**Figure 5.59 Jet penetration, PR=5**

Figure 5.59 shows the trajectory of the center streamline of the jet for PR=5.0 issuing into a subsonic and supersonic freestream along with the PR=1.5 jet issuing into a $M=0.3$ flow. Increasing the freestream Mach number dramatically decreased jet penetration compared to the subsonic cases. At a distance of 40 diameters downstream of the jet exit, the jet penetrated a mere 3 diameters into the Mach 2 flow while the same jet pressure ratio penetrated six times further (almost 19 diameters) into the Mach 0.3
freestream. The lack of penetration by the underexpanded jet in Mach 2 flow prevented the formation of the downstream pair of global separation lines (like those seen in Figure 5.43) and the formation of near- or far-field wake vortices. With no near-field wake vortices and the new pair of global separation lines defining a separation zone much larger than what could be reasonably called “near field”, it was viewed as more reasonable to define the near field by the judicious selection of a surface pressure contour as shown in Figure 5.60.

![Flow Pressure contours Near field boundary](image)

**Figure 5.60 Near field boundary, $M=2.0$, PR=5**

Figure 5.60 shows pressure contours in white with the near field boundary in bold red. The shape of the pressure contours in this simulation was radically different from the subsonic simulation and covered a much larger area. The contours took a more mushroom-like shape with the two lobe pattern morphing into the stem of the mushroom.
and the upstream contours resembling the top of a mushroom. The presence of shock and expansion waves in the Mach 2 cases altered the shape of the pressure contours as well as the separation topology and with the near field being redefined by the contours instead of the topology, transverse jets in supersonic flow (TJISF) had to be considered a completely different class of flow fields from their subsonic counterparts. Examination of the surface pressure distribution illustrated further the difference between these two classes of flow. The upstream distributions are shown in Figure 5.61 and the downstream distributions in Figure 5.62.

Figure 5.61 Upstream surface pressure at \( z/d_{jet} = 0.0 \), PR=5
Figure 5.61 shows normalized surface pressure plotted against normalized axial distance upstream of the jet at $z/d_{jet}=0.0$ for the PR=5.0 jet issuing into both subsonic ($M=0.3$) and supersonic ($M=2.0$) freestream with the edge of the jet highlighted for reference. The shock waves upstream of the jet elevated the surface pressures 80% above the subsonic case and enlarge the affected area. This marked difference in the upstream region had an amplifying effect on the jet thrust while the downstream surface pressures attenuated the thrust as shown in Figure 5.62 where surface pressure downstream of the jet are plotted. The jet expansion waves reduced the surface pressure downstream of the jet by 60% from the subsonic case and enlarged the affected downstream area. Although these distributions show the jet thrust was amplified by the upstream region and attenuated by the downstream, the area affected was three-dimensional and integration of the entire area was required to determine the global impact of jet interaction in supersonic flow on jet thrust.
Table 5.4 summarizes the thrust coefficient, force coefficient and moment coefficient along with the amplification coefficients for the supersonic and subsonic simulations at jet pressure ratio 5.

Table 5.4 Coefficients For Pressure Ratio 5.0

<table>
<thead>
<tr>
<th>Mach Number</th>
<th>Pressure Ratio</th>
<th>$C_{N_{ji}}$</th>
<th>$C_{m_{ji}}$</th>
<th>$C_T$</th>
<th>$\varepsilon_N$</th>
<th>$\varepsilon_m$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.3</td>
<td>5.0</td>
<td>+1.54e-3</td>
<td>+7.79e-4</td>
<td>-1.64e-3</td>
<td>+0.06</td>
<td>-0.22</td>
</tr>
<tr>
<td>2.0</td>
<td>5.0</td>
<td>-5.77e-5</td>
<td>-2.28e-5</td>
<td>-3.65e-5</td>
<td>+2.58</td>
<td>+2.60</td>
</tr>
</tbody>
</table>

The state of the freestream changed the direction of the jet interaction force and moment, $C_{N_{ji}}$ and $C_{m_{ji}}$, and, because of the difference in dynamic pressure, reduced the thrust coefficient by two orders of magnitude. The result was that the thrust and moment were amplified by approximately 160%. This marked difference in amplification coefficients is a direct consequence of the change in character of the flow structure between subsonic and supersonic freestream.

In the subsonic case, the jet interaction force was nearly equal to the jet thrust and opposite in direction implying the low pressure region aft of the jet generated a force larger than the high pressure upstream region and the jet thrust combined, therefore nullifying the jet thrust. By contrast, the integration of surface pressures for the supersonic case showed the jet interaction force was more than 150% greater than the jet thrust, but in the same direction. This implied that the upstream region, dominated by the $\lambda$ shock, overwhelmed the region aft of the jet and amplified the jet thrust. The change in character of the flow structure between subsonic and supersonic freestreams resulted in surprisingly different amplification coefficients, surface pressure distributions and jet
penetration and underlined the importance of understanding jet interaction forces and the flow structure that produced them.

The root cause of the thrust amplification was the $\lambda$ shock structure which caused the high pressure region upstream of the jet. Further investigation of the $\lambda$ shock and the near field flow structure was done in the following section. In the following section, the impact of jet PR on flow structure, surface pressures and jet interaction amplification coefficients is examined by discussing the results of simulations for five different PR.

5.1.2.2 Pressure Ratio Variation

In this section, transverse jets with various pressure ratios, namely 15, 100, 500, 1000 and 2000, issuing into a Mach 2 freestream are examined. The same flat plate and nozzle configuration used in the previous calculations were used with the nozzle inlet total pressures listed in Table 5.5.

<table>
<thead>
<tr>
<th>PR</th>
<th>$P_{t_{jet}}$ (kPa)</th>
<th>$P_{t_{jet}}$ (psf)</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>147.5</td>
<td>3080.4</td>
</tr>
<tr>
<td>100</td>
<td>983.3</td>
<td>20535.9</td>
</tr>
<tr>
<td>500</td>
<td>4916.3</td>
<td>102679.4</td>
</tr>
<tr>
<td>1000</td>
<td>9832.6</td>
<td>205358.8</td>
</tr>
<tr>
<td>2000</td>
<td>19665.2</td>
<td>410717.6</td>
</tr>
</tbody>
</table>

Table 5.5 Nozzle Inlet Total Pressures

Figure 5.63 shows streamlines through the nozzle for each of these conditions.
The progressively increasing jet pressure ratio had two noticeable effects: greater jet penetration and stronger shocks and larger expansion fans in both the jet and the freestream. The upstream interaction between these jets and the Mach 2 freestream were similar to the PR=5 calculations as shown in Figures 5.64 through 5.68.
Figure 5.64 Upstream pressure contours and velocity vectors, $M=2.0$, PR=15

Figure 5.65 Upstream pressure contours and velocity vectors $M=2.0$, PR=100
Figure 5.66 Upstream pressure contours and velocity vectors $M=2.0$, PR=500

Figure 5.67 Upstream pressure contours and velocity vectors $M=2.0$, PR=1000
Figure 5.68 Upstream pressure contours and velocity vectors $M=2.0$, PR=2000

As the jet pressure ratio increased, the obstruction of the freestream by the jet moved the shock structure upstream, strengthened the bow shock, enlarged the interaction region and sharpened the barrel shock. Furthermore, the recirculation regions grew and the stagnation point moved upstream and the extent of the influence of the upstream flow structure as jet pressure ratio increased is shown in Figure 5.69.
Figure 5.69 shows the upstream surface pressure distributions for each PR examined with the PR=5 shown for comparison. The character of these distributions is similar and directly related to the upstream flow structure illustrated in Figure 5.70.
Figure 5.70 Upstream flow structure with surface pressure, $M=2.0$, PR=500

The features shown in Figure 5.70 illustrate the complexity of the interaction between the freestream, the approaching boundary layer and the jet. The figure relates the flow structure to the character of the surface pressure distribution. The initial pressure rise in the distribution was produced by the oblique compression waves of the $\lambda$ shock followed by a relatively flat distribution. The pressure depression here is the result
of a vortical core. After this dip, the pressure rose to a peak at the stagnation point (discussed previously in subsection 5.1.2.1 and shown in Figure 5.52). Following the stagnation point, another dip was realized because of the presence of a second vortical structure. This vortical structure was the origin of a second pair of dividing surfaces. This pair of dividing surfaces was generated by the combination of the second upstream saddle point and the upstream attachment node. As with the first pair of dividing surfaces, a depression occurred as a result of the second vortical core. With the presence of the second saddle point (see Figure 5.70) and the associated global separation lines, another $\lambda$ shock structure was formed. As the jet exited the nozzle, it expanded supersonically toward the stagnation point via a series of oblique waves that began to form. As the jet approached the stagnation point, the flow separated from the surface forming the second saddle point. As this saddle point formed, the oblique waves coalesced into the barrel shock in a manner similar to the oblique waves coalescing into the bow shock. Similar to the bow shock, the barrel shock was refracted through the viscous layer (see Figure 5.54) and turned the flow toward the node of attachment thereby creating a second recirculation region and $\lambda$ shock structure within the separation zone. The oblique compression waves associated with the second $\lambda$ shock structure caused the pressure to rise after the second dip in the pressure distribution while the second pair of global lines of separation marked the end of the upstream separation zone. Downstream of these separation lines, the oblique waves and jet expansion dictated the pressure distribution. Figures 5.71 through 5.75 show skin friction lines in black and
surface pressure contours in white with separation topology highlighted for each PR except PR=5 which was shown previously in Figure 5.56.

Figure 5.71 Skin friction lines with pressure contours, $M=2.0$, PR=15
Figure 5.72 Skin friction lines with pressure contours, $M=2.0$, PR=100

Figure 5.73 Skin friction lines with pressure contours, $M=2.0$, PR=500
Figure 5.74 Skin friction lines with pressure contours, $M=2.0$, PR=1000

Figure 5.75 Skin friction lines with pressure contours, $M=2.0$, PR=2000
As these figures are compared, the changing size of the separated zone and the near field are apparent when recognizing that the jet exit shrinks as the PR increased. As the PR increased, the near field also increased. As the near field grew, bifurcations continued up to PR 100. At PR=15, a pitchfork bifurcation caused the downstream saddle point to change into two saddle points and an attachment node. At PR 100, another pitchfork bifurcation (Chapman 1986) occurred changing two separation foci and a saddle point into a single separation node. At PR greater than 100, the phase portrait remained constant. The bifurcation between PR=5 and 15 added another pair of global separation lines and near and far field wake vortices while the bifurcation between 15 and 100 eliminated the horn vortices, but added a second pair of horseshoe vortices.

Figure 5.76 First horseshoe vortices, PR=15
Figure 5.77 First horseshoe vortices, PR=100

Figure 5.78 First horseshoe vortices, PR=500
Figure 5.79 First horseshoe vortices, PR=1000

Figure 5.80 First horseshoe vortices, PR=2000
Figure 5.76 through 5.80 shows streamlines for each PR wrapping around the first pair of dividing surfaces to form horseshoe vortices around the jet with skin friction lines in black and surface pressure contours in white for reference. As streamlines passed the global lines of separation, the streamlines wrap under each other in a manner similar to the PR=5 calculations (see Figure 5.60) with the left running vortex rotating \textit{clockwise} and right running vortex rotating \textit{counterclockwise} as they convected downstream. As PR increased, the vortices grew in size. Figure 5.81 plots a streamline along the centerplane of the flow domain for PR=100, 500 and 1000 to illustrate the growth of the horseshoe vortices with PR.

![Figure 5.81 Center streamline of first horseshoe vortices](image)

As the PR increased, the gradient intensified creating larger and stronger horseshoe vortices moving upstream away from the jet.
Figure 5.82 Second horseshoe vortices, PR=15

Figure 5.83 Second horseshoe vortices, PR=100
Figure 5.84 Second horseshoe vortices, PR=500

Figure 5.85 Second horseshoe vortices, PR=1000
Figure 5.86 Second horseshoe vortices, PR=2000

Figure 5.82 through 5.86 shows streamlines for each PR wrapping around the second pair of dividing surfaces thereby forming another pair of horseshoe vortices around the jet. Skin friction lines are shown in black and surface pressure contours in white. The dividing surfaces emanating from the second pair of global separation lines were different from the first pair of dividing surface in two ways: first, the left running vortex rotated counterclockwise, and second, they were taller, as illustrated in Figure 5.87.
Figure 5.87 is an illustration of the upstream dividing surfaces with streamlines sketched and singularities identified for reference. In an arrangement typical of the pressure ratios examined, there were three (3) singularities on the surface and three (3) singularities in the flow.

The relative position of the singularities shown in Figure 5.87 to each other was the same for each pressure ratio examined.
Figure 5.88 shows the position of upstream singularities for each PR. As PR increased, the singularities moved upstream, but their position relative to each other remained fairly constant. The attachment node was between the two (2) saddle points and there was always one interior singularity upstream of the node and two (2) downstream. Of the three (3) singularities within the flow, two (2) were origins for the vortical cores of dividing surfaces. The third was positioned above and downstream of the attachment node and above and upstream of the vortical core of the second pair of horseshoe vortices. The position of this singularity dictated the height of the second pair of dividing surfaces and allowed the freestream to enter into the second pair of horseshoe vortices while preventing the jet flow from entering the vortices. Thus, the arrangement of the singularities prevented the mixing of freestream and jet in either pair of horseshoe vortices.
However, since the rotation of these vortices was in the same sense as the jet vortices, as they wrapped around the jet, they became entrained into the jet vortices as shown in Figures 5.89 through 5.93.

Figure 5.89 Jet vortices, PR=15
Figure 5.90 Jet vortices, PR=100

Figure 5.91 Jet vortices, PR=500
These figures show the streamlines in the second horseshoe vortices from Figure 5.82 through 5.86 with the streamlines from around the circumference of the jet. Although
these figures show that the horseshoe vortices became entrained into the jet flow, the position of the singularity and the presence of the dividing surfaces created a barrier between the jet and the freestream keeping the flows separated as they convected downstream within the jet plume. The streamlines in Figures 5.89 through 5.93 show there was essentially no intermingling between the jet vortices and the second horseshoe vortices even after the horseshoe vortices were entrained into the jet plume. More pertinent to the surface pressures and JI forces were the emergence of the second pair of horseshoe vortices, the elevation of the surface pressure at the stagnation point upstream of the jet and the fundamental changes in separation topology downstream of the jet.

The pitchfork bifurcation downstream split the saddle point into two saddle points, a attachment node and another pair of global separation lines. These changes in separation topology caused the reemergence of the near- and far-field wake vortices with the near field wake vortices shown in Figures 5.94 through 5.98 from each PR.
Figure 5.94 Near field wake vortices, PR=15

Figure 5.95 Near field wake vortices, PR=100
Figure 5.96 Near field wake vortices, PR=500

Figure 5.97 Near field wake vortices, PR=1000
The new attachment node was positioned between the two new global separation lines downstream of the separation nodes and was the dividing point between the near and far field wake vortices. The flow was ahead of the attachment node and the downstream saddle points was carried by the dividing surfaces upstream toward the jet with the left running vortex rotating clockwise and the right running vortex rotating counterclockwise until they turned vertically at the separation node and became entrained in the jet as shown in Figures 5.94 though 5.98. A comparison of surface pressure contours between PR=5 (Figure 5.60) and PR=15 (Figure 5.71) in this region shows the presence of the near-field wake vortices widened the defect in the contours while streamlines (not shown) from the vortices appeared to have intermingled with the jet flow after entrainment making the near field wake vortices the best hope for mixing the jet and the freestream, however desirable or limited that might be. So with the emergence of the
downstream attachment node, the near field gained a limited ability to mix the jet and freestream and forced a limited change in the surface pressure distribution.

If flow was downstream of the attachment node, the left running vortex rotated *clockwise* convecting downstream away from the jet and the right running vortex rotated *counterclockwise* downstream as shown in Figure 5.99 and 5.103.

![Diagram showing far field wake vortices](image)

**Figure 5.99** Far field wake vortices, PR=15
Figure 5.100 Far field wake vortices, PR=100

Figure 5.101 Far field wake vortices, PR=500
Figure 5.102 Far field wake vortices, PR=1000

Figure 5.103 Far field wake vortices, PR=2000
These figures show streamlines wrapping around the dividing surface emanating from the downstream global lines of separation with skin friction lines shown in black and the surface pressure contours shown in white. Even though there were no wake vortices in the PR=5 calculations, there was a defect in the surface pressure contours in that area and it widened at PR=100 as the far-field wake vortices reappeared and grew as PR increased.

Wave formation resulting from the presence of the far-field wake vortices along with jet expansion dictated the character of the downstream pressure distributions.

![Figure 5.104 Downstream surface pressure, M=2.0](image)

Figure 5.104 shows the downstream surface pressure distribution for each PR examined. The character of these distributions was quite different from the upstream distributions. The general character of the downstream surface pressure distributions can be described as an overexpansion followed by a gradual recompression. Figure 5.105
shows the downstream surface pressure distribution along the centerplane for PR=1000 and pressure contours on the centerplane with the pertinent wave formations highlighted.

Figure 5.105 Downstream flow structure with surface pressure, $M=2.0$, PR=1000

The downstream flow structure included the barrel shock and expansion fan similar to that shown previously in Figure 5.70, as well as the separation and attachment
nodes and a shock reflection. In addition, a series of compression waves generated by the
growth of the far-field wake vortices and a Mach disk were present, but the structure with
the largest influence on the character of the downstream pressure distribution was the
expansion of the jet. The jet overexpands due to the turning and realignment of the jet
flow through the series of expansion and compression wave intersections and reflections
previously discussed. With the jet overexpansion, the surface pressure dropped to near
vacuum, creating a separation node and, in very close proximity, an attachment node
where the growth of the far field wake vortices began. As the vortices grew, compression
waves formed, eventually coalescing into the shock reflected from the Mach disk. The
compression waves provided the gradual pressure rise seen in the downstream pressure
distribution. The shock reflected from the Mach disk traveled downstream impinging on
the surface and changing the slope of the surface pressure distribution and causing the
surface pressures to overshoot the ambient. The impingement was followed by a series
of expansion waves resulting from the lateral spreading of the far field wake vortices
bringing the surface pressure to ambient well downstream of the jet.

The extent of the downstream influence of JI perturbed surface pressures to the
edge of the flow domain as shown in Figure 5.106.
Figure 5.106 Near field boundary, PR=2000

Figure 5.106 shows the surface pressure contours with selected contours identified. The area significantly affected by jet interaction at PR=2000 extended 110 diameters downstream and was 180 diameters wide. It was viewed as unreasonable to define an area of such extent as “near field”, but the integration of surface pressures would yield an incorrect force if this whole area were not included. With no other recourse, the edge of the plate was used as part of the “near field” boundary if it were warranted. Otherwise, the judicious selection of a pressure contour was used as shown in Figure 5.107 which compares the “near field” for each PR.
Figure 5.107 shows the growth of the near field as PR increased. As PR increased, both the upstream high pressure region and the downstream low pressure region increased in size and although the maximum pressure upstream of the jet increased and the minimum pressure downstream decreased (shown in Figure 5.71 and 5.98), the dramatic change in the downstream area offset the pressure increase upstream and caused the amplification coefficients to drop. Table 5.6 summarizes the coefficients for all pressure ratios investigated.
Table 5.6 Coefficients For Pressure Ratio Sweep, $M=2.0$

<table>
<thead>
<tr>
<th>Pressure Ratio (PR)</th>
<th>$C_{Nj}$</th>
<th>$C_{mji}$</th>
<th>$C_T$</th>
<th>$\varepsilon_N$</th>
<th>$\varepsilon_m$</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>-5.77e-5</td>
<td>-2.28e-5</td>
<td>-3.66e-5</td>
<td>2.58</td>
<td>2.60</td>
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<tr>
<td>15</td>
<td>-1.43e-4</td>
<td>-5.74e-5</td>
<td>-1.28e-4</td>
<td>2.12</td>
<td>2.16</td>
</tr>
<tr>
<td>100</td>
<td>-7.80e-4</td>
<td>-2.73e-4</td>
<td>-9.03e-4</td>
<td>1.86</td>
<td>1.78</td>
</tr>
<tr>
<td>500</td>
<td>-2.64e-3</td>
<td>-2.54e-4</td>
<td>-4.56e-3</td>
<td>1.58</td>
<td>1.14</td>
</tr>
<tr>
<td>1000</td>
<td>-3.86e-3</td>
<td>+7.25e-4</td>
<td>-9.13e-3</td>
<td>1.42</td>
<td>0.80</td>
</tr>
<tr>
<td>2000</td>
<td>-3.95e-3</td>
<td>+3.87e-3</td>
<td>-1.83e-2</td>
<td>1.22</td>
<td>0.46</td>
</tr>
</tbody>
</table>

The increase in jet pressure ratio gradually increased the magnitude of the jet interaction force and moment coefficients, $C_{Nj}$ and $C_{mji}$, as well as the jet thrust. The moment changed direction as the center of pressure moved upstream of the leading edge while the amplification coefficients decreased. JI amplified the jet thrust at all PR considered while the jet moment became attenuated at PR greater than 500. Figure 5.108 shows the trend of both amplification coefficients for PR.

![Figure 5.108 Amplification coefficients, $M=2.0$](image-url)
Between PR 5 and 100, when the bifurcations occurred, there were sharp changes in the amplification coefficients. After the topology stabilized, the force amplification coefficient correlated with PR according to the quadratic relation,

$$PR = 4099.1e_N^2 + 15515e_N + 14778$$

and the exponential relation,

$$PR = 5917.1e^{-2.26e_N}$$

for the moment amplification coefficient. The agreement between the correlating parameter and the data is shown in Figure 5.109.

Figure 5.109 Amplification coefficient correlations, $M=2.0$

Figure 5.109 shows the amplification coefficients plotted against PR with correlations for both force and moment plotted as well. In addition to correlation with PR, this figure shows that the force and moment amplification coefficients are not 1.0 at the same PR. For reaction jet control systems operating in endo-atmospheric flight, it would be desirable to operate the system at a PR where the amplification coefficient for
both was 1.0 so jet interaction could be ignored. However, this data for a flat plate in Mach 2 flow shows there was no PR where the force and moment amplification coefficients were both 1.0.

5.1.2.3 Mach Number Variation

In this section, a transverse jet of pressure ratio 2000 issuing into various supersonic flows at freestream Mach numbers of 2.5, 3.0, 3.5 and 4.0 is compared to the Mach 2.0 case to determine the effect of freestream Mach number on the near-field flow structure and JI forces. The PR=2000 was selected because typical applications of RJCS have chamber pressures of 3000 psia while at altitudes with ambient pressures of 1.5 psia. The same flat plate and nozzle configuration used in the previous calculations were used. To maintain the same Reynolds number and jet pressure ratio, both the ambient and nozzle inlet total pressure were modified for each freestream Mach according to Table 5.7.

Table 5.7 Flow Conditions, PR=2000

<table>
<thead>
<tr>
<th>Mach Number (M)</th>
<th>$p_\infty$ (KPa)</th>
<th>$p_\infty$ (psf)</th>
<th>$p_{jet}$ (KPa)</th>
<th>$p_{jet}$ (psf)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.5</td>
<td>7.87</td>
<td>164.3</td>
<td>15732</td>
<td>328574</td>
</tr>
<tr>
<td>3.0</td>
<td>6.55</td>
<td>136.9</td>
<td>13110</td>
<td>273812</td>
</tr>
<tr>
<td>3.5</td>
<td>5.62</td>
<td>117.4</td>
<td>11237</td>
<td>234696</td>
</tr>
<tr>
<td>4.0</td>
<td>4.92</td>
<td>102.7</td>
<td>9833</td>
<td>205359</td>
</tr>
</tbody>
</table>

Figure 5.110 shows streamlines through the nozzle for each of these conditions.
Figure 5.110 Nozzle streamlines, Mach sweep

Figure 5.110 shows that increasing freestream Mach number had little effect on jet streamlines. Compared with the $M=2.0$ calculations (see Figure 5.63), the jet streamlines were essentially unchanged when freestream Mach increased. The wave formations were very similar between the Mach numbers examined both upstream and downstream as shown in Figures 5.111 through 5.120.
Figure 5.111 Upstream pressure contours and velocity vectors, $M=2.0$

Figure 5.112 Upstream pressure contours and velocity vectors, $M=2.5$
Figure 5.113 Upstream pressure contours and velocity vectors, $M=3.0$

Figure 5.114 Upstream pressure contours and velocity vectors, $M=3.5$
Figure 5.115 Upstream pressure contours and velocity vectors, $M=4.0$

Figure 5.116 Downstream pressure contours and velocity vectors, $M=2.0$
Figure 5.117 Downstream pressure contours and velocity vectors, $M=2.5$

Figure 5.118 Downstream pressure contours and velocity vectors, $M=3.0$
Figures 5.111 through 5.115 show velocity vectors and pressure contours on the centerplane of the flow domain upstream of the jet for each freestream Mach number.
examined while Figures 5.116 through 5.120 show velocity vectors and pressure contours downstream of the jet. Both upstream and downstream of the jet, the same wave formations were present at each Mach number. As the Mach number increased, the bow shock moved downstream and became more oblique, the first \( \lambda \) shock structure moved downstream and became more oblique, the interaction region shrank and the first upstream recirculation shrank while the location of the stagnation point moved downstream and the size of the second recirculation and \( \lambda \) were reduced. Downstream of the jet, as the Mach number increased, the barrel shock became more inclined similar to the upstream bow shock while the Mach disk, compression wavelets and shock reflection moved upstream, not downstream. This behavior was a result of the upstream bow shock, becoming more oblique which raised the Mach number downstream of the shock and reduced the total pressure loss. With relatively higher pressures downstream, the wave forms moved upstream, instead of downstream.

Although the wave structure both upstream and downstream were the same, the position of the structure changed with increasing Mach number indicating a change in separation location as shown in Figures 5.121 through 5.124.
Figure 5.121 Skin friction lines with pressure contours, $M=2.5$

Figure 5.122 Skin friction lines with pressure contours, $M=3.0$
Figure 5.123 Skin friction lines with pressure contours, $M=3.5$

Figure 5.124 Skin friction lines with pressure contours, $M=4.0$
These figures show skin friction lines in black and surface pressure contours in white for each Mach number examined. The separation topology was the same as the topology seen in section 5.1.2.2 for Mach 2.0 calculations (see Figure 5.75) with four saddle points, two upstream and two downstream, three nodes, one attachment upstream, one attachment downstream, one separation downstream, two pairs of global separation lines around the jet dividing the freestream, the jet and the separation zone and one pair of global separation lines downstream. With the same separation topology, all the same vortices were present as well; two pairs of horseshoe vortices, a pair of near and far field wake vortices and jet vortices.

The shock structure responsible for the presence of the vortices had a significant influence on the surface pressure as discussed in the previous section and shown in the surface pressure distributions along the centerplane in Figures 5.125 and 5.126.

![Figure 5.125 Upstream centerline surface pressures, PR=2000](image)
Figure 5.125 shows the normalized surface pressure distribution upstream of the jet for each Mach number with Mach 2.0 shown for comparison. As seen in the pressure contours of Figure 5.68 and Figures 5.111 through 5.115, Figure 5.125 shows the initial pressure rise moved downstream until Mach 3.5 where it remained unchanged for Mach 4.0. The upstream flow structure discussed in section 5.1.2.2 and shown in Figure 5.70 apply to these distributions as well with the only difference being the position of the wave formations which resulted in higher surface pressures upstream the jet.

Figure 5.126 shows the normalized surface pressure distribution downstream of the jet for each Mach number considered with Mach 2.0 shown for comparison. As seen in the pressure contours of Figures 5.116 through 5.120, Figure 5.126 shows the overshoot in the pressure distribution moved upstream then disappeared at Mach 4.0. As Mach number increased, the area affected the wave formation downstream of the jet shrank as shown in the surface pressure contour plots in Figures 5.127 through 5.130.
Figure 5.127 Surface pressures, $M=2.5$

Figure 5.128 Surface pressures, $M=3.0$
Figure 5.129 Surface pressures, $M=3.5$

Figure 5.130 Surface pressures, $M=4.0$
Figure 5.127 through 5.130 shows surface pressure contours around the jet with selected pressure levels highlighted. As seen previously, the contours had a mushroom-like shape and shrank as Mach number increased leading to smaller near field boundaries as shown in Figure 5.131.

![Figure 5.131 Near field boundaries, PR=2000](image)

Figures 5.106 and 5.127 through 5.130 show surface pressure contours normalized by the ambient pressure for each Mach number. As the Mach increases, the upstream high pressure region and the downstream low pressure region decrease in size until Mach 3.5 when the shape of the surface pressure contours for Mach 3.5 and 4.0 (Figures 5.129 and 5.130) are essentially the same. The high pressure region maintains a
reasonable mushroom shape while the maximum normalized pressure increases by approximately 50% from Mach 2.0 to 4.0. The low pressure region maintains symmetry about the centerplane and shrinks while the minimum pressure remains constant. The character of the flow structure is similar for all Mach numbers, but the changes in area both upstream and downstream and in the magnitude of the upstream pressure result in increased amplification coefficients with increasing Mach number. Table 5.8 summarizes the coefficients for all Mach numbers investigated.

<table>
<thead>
<tr>
<th>Mach Number (M)</th>
<th>Pressure Ratio (PR)</th>
<th>$C_{Nji}$</th>
<th>$C_{mji}$</th>
<th>$C_T$</th>
<th>$\epsilon_N$</th>
<th>$\epsilon_m$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.0</td>
<td>2000</td>
<td>-3.95e-3</td>
<td>+3.87e-3</td>
<td>-1.83e-2</td>
<td>+1.22</td>
<td>+0.46</td>
</tr>
<tr>
<td>2.5</td>
<td>2000</td>
<td>-9.52e-3</td>
<td>-3.40e-3</td>
<td>-1.17e-2</td>
<td>+1.81</td>
<td>+1.75</td>
</tr>
<tr>
<td>3.0</td>
<td>2000</td>
<td>-8.79e-3</td>
<td>-4.15e-3</td>
<td>-8.10e-3</td>
<td>+2.08</td>
<td>+2.32</td>
</tr>
<tr>
<td>3.5</td>
<td>2000</td>
<td>-7.90e-3</td>
<td>-4.03e-3</td>
<td>-5.96e-3</td>
<td>+2.32</td>
<td>+2.74</td>
</tr>
<tr>
<td>4.0</td>
<td>2000</td>
<td>-7.42e-3</td>
<td>-3.94e-3</td>
<td>-4.57e-3</td>
<td>+2.62</td>
<td>+3.22</td>
</tr>
</tbody>
</table>

The increase in freestream Mach number from 2.0 to 2.5 more than doubled the jet interaction force, $C_{Nji}$, while as the freestream Mach increased above 2.5, $C_{Nji}$ gradually decreased. The jet interaction moment, $C_{mji}$, remained fairly constant over the Mach range considered with the center of pressure moving downstream as freestream Mach increased from 2.0 to 2.5. The magnitude of the jet thrust decreased with increasing freestream Mach because the total pressure of the jet decreased as the freestream Reynolds number was held constant at $6.56 \times 10^6$/m. The trend of $C_{Nji}$ indicates there is a maxima between Mach 2.0 and 2.5 and the trend in $C_{mji}$ indicates there is a zero moment about the leading edge between Mach 2.0 and 3.0.
Despite the trends in $C_{Nji}$ and $C_{mji}$, the amplification coefficients for both the jet interaction force and moment increase with increasing Mach number as shown in Figure 5.132.

![Figure 5.132 Amplification coefficients from PR and Mach number sweeps](image)

Figure 5.132 shows the force and moment amplification coefficients from both the pressure ratio variation (discussed in section 5.1.2.2) and the Mach number sweep plotted against the momentum flux ratio ($J$) on a log-log scale. The PR sweep was shown previously in Figure 5.108 and the Mach sweep shows as momentum flux ratio increases the amplification of the jet thrust decreases, but is always amplifying the jet thrust. The amplification coefficient for the moment however attenuates the jet moment at $J$~100.

As seen in this figure, the amplification coefficients for the PR and M variations do not coincide. This indicates that the momentum flux ratio does not correlate these coefficients very well. In an effort to correlate the parametric data, a modification to the momentum flux ratio was made and the data re-plotted as shown in Figure 5.133. Figure
5.133 shows the amplification coefficients for both the pressure ratio and Mach sweeps plotted against the modified momentum flux ratio which was defined as

$$J^* = \left( \frac{p_{ej}}{p_{\infty}} \right)^{0.2} \left( \frac{M_{ej}}{M_{\infty}} \right)^{2.2}$$

The data is plotted on a linear-log scale and both variations correlate well using this parameter. The use of this modified parameter shows the relative importance of jet pressure ratio to the Mach number ratio. Since the Mach number ratio is squared and the pressure ratio raised is to the 1/5th power, it can be concluded the Mach number ratio has a greater influence over the amplification and attenuation of the jet thrust than the pressure ratio.
5.2 Transverse Jet From A Body of Revolution

5.2.1 Convergent Nozzle

In this section, a transverse jet of PR 2000 is examined that issues from a cylindrical body preceded by a 3:1 von Kármán nose with a spherical tip through the convergent nozzle shown in Figure 5.2. The near-field flow structure resulting from this jet issuing into a supersonic (M=2) freestream is investigated and compared to the flat plate simulation discussed in section 5.1.2.2. The geometry of this body is shown in Figure 5.134.

Comparison between the FP and BOR highlights the effect of body geometry on the near-field mean flow structure of transverse jets in supersonic freestreams. Figure 5.135 shows streamlines exiting the nozzle of the BOR with streamlines through the nozzle of the flat plate (FP) shown for reference.
These streamlines spread in a similar manner to the flat plate (FP), but with slightly more downstream inclination. This similarity implied similar wave formations upstream of the jet as shown in Figure 5.136 and 5.137, modified by the transverse curvature afforded by the body of revolution. Figure 5.136 shows velocity vectors and pressure contours on the centerplane of the flat plate flow domain with the jet PR of 2000 issuing into a Mach 2 freestream and Figure 5.137 shows velocity vectors and pressure contours on the centerplane of the BOR.
Figure 5.136 Upstream pressure contours and velocity vectors, FP

Figure 5.137 Upstream pressure contours and velocity vectors, BOR
The basic flow structures for both the FP and BOR were similar, including the bow shock, an interaction region, the barrel shock, a $\lambda$ shock and a stagnation point. However, Figure 5.137 shows the bow shock was normal away from the surface while the bow shock in Figure 5.136 was curved. This difference was due to the shape of the jet plume obstructing the freestream which was dictated by the size of the $\lambda$ shock. The $\lambda$ shock was smaller in the BOR case because of the favorable pressure gradient on the nose reducing the height of the $\lambda$ and altering its effect on the plume. Moreover, the BOR had only one upstream recirculation and no saddle point in the interaction region because of the stagnation point at the tip of the nose. The upstream recirculation resulted from the combination of the attachment and the first saddle point in the near field. The saddle point closer to the jet acted in combination with the attachment node at the nose tip. With this saddle point downstream of its attachment node, the pressure gradient was favorable so no recirculation was generated. These differences in flow structure had only a minor impact on the character in upstream pressure distribution as shown in Figure 5.138.
Figure 5.138 shows normalized surface pressures plotted against the normalized axial distance upstream of the jet for both the BOR and FP. By comparing the distributions at equivalent upstream locations, it is seen the initial pressure rise occurred further downstream on the body than the flat plate. The positive pressure gradient on the nose of the body allowed the boundary layer to penetrate further into the adverse pressure gradient created by the jet than the flat plate boundary layer. After the initial pressure rise, caused by the oblique wavelets of the $\lambda$ shock, there was a slight depression (described previously in section 5.1.2.2) caused by the recirculation followed by a peak at the attachment node and an expansion thereafter into the barrel shock. The expansion was interrupted by a small pressure rise as a result of the curvature of the streamlines as they turned upward due to the obstruction of the jet. This was different than the flat plate as discussed in section 5.1.2.2 yet it yielded the same character in the pressure distribution. It can be seen in Figure 5.138 that the stagnation pressure and the surface
pressures downstream of it were significantly higher than the FP case covering a smaller area. This had a significant impact on the amplification coefficient and the flow structure in the near field, but the separation topologies were very similar.

Figure 5.139 Separation topology, BOR, convergent nozzle, $M=2$

Figure 5.139 shows skin friction lines with surface pressure contours in white for the BOR. Upstream of the jet, there were two saddle points and one attachment node with two pairs of global separation lines around the jet. The first pair of global separation lines wrapped around the body without converging to a node of separation. Downstream of the jet there were two saddle points, a node of separation and a node of attachment for a total of four saddle points and three nodes however, a fourth node not shown in Figure 5.139 was present at the tip of the nose and it was reasoned that the first pair of global separation lines terminated at two nodes of separation at the base of the body so there were six nodes with four saddle points in this topology classifying this flow as a three-dimensional body in uniform flow.
Although the topologies were similar, the addition of an attachment node at the tip of the nose altered the flow structure by eliminating the second pair of horseshoe vortices leaving only one pair of horseshoe vortices in the BOR calculations. This pair of horseshoe vortices were generated in the same manner as in the FP calculations with the upstream saddle point and attachment node acted in combination to generate this pair of horseshoe vortices as shown in Figure 5.140.

![Horseshoe vortices](image)

Figure 5.140 Horseshoe vortices, BOR, convergent nozzle, $M=2$

This figure shows streamlines wrapping around the dividing surfaces of the upstream global separation lines forming horseshoe vortices with skin friction lines in black and surface pressure contours in white for reference. These horseshoe vortices were similar to the FP horseshoe vortices with the left running vortex rotating clockwise, except that these vortices wrapped around the body perturbing the surface pressures on the side of the body as well as the upper surface.
In contrast to the FP cases, a second pair of horseshoe vortices did not form for the BOR case. The second pair of horseshoe vortices was eliminated by the elimination of the second \( \lambda \) shock. The favorable pressure gradient created by the attachment node at the nose tip created a sufficient pressure rise to preclude the jet flow from forming a second \( \lambda \), a second recirculation and a second pair of horseshoe vortices. In addition to the elimination of the second pair of horseshoe vortices, the near-field wake vortices were so small, they were considered negligible, having no effect on the near field mean flow structure, surface pressures or the JI force and moment.

However, the far-field wake vortices were not negligible as shown in Figure 5.141.

![Figure 5.141 Far field wake vortices, BOR, convergent nozzle, M=2](image)

Figure 5.141 Far field wake vortices, BOR, convergent nozzle, M=2
Like the FP calculations, the far-field wake vortices were present because of the downstream global separation lines. Figure 5.141 shows streamlines wrapping around the dividing surfaces emanating from these lines. Similarly with the FP cases, the far-field wake vortices were a pair of counter rotating vortices convecting downstream separated by the symmetry plane with the left running vortex rotating clockwise and the effect of the far field wake vortices was the same as the FP cases generating compression waves and a gradual recovery of the surface pressures. Comparison of the downstream wave formations in Figures 5.142 and 5.143 shows the compression waves formed by the growth of the far-field wake vortices.

Figure 5.142 Downstream pressure contours and velocity vectors, FP
Figure 5.143 Downstream pressure contours and velocity vectors, BOR

Figure 5.142 shows velocity vectors and pressure contours on the centerplane of the flat plate flow domain downstream of the jet and Figure 5.143 shows velocity vectors and pressure contours on the centerplane of the BOR downstream of the jet. Figure 5.143 shows the same flow structure as the FP with a barrel shock, jet expansion fan, Mach disk, compression wavelets and far field wake vortices as well as a separation and attachment nodes.
Figure 5.144 shows that the difference in downstream surface pressure distribution between the FP and BOR is small. Both follow the same trend with nearly the same magnitudes up to ambient pressure where the BOR distribution overshoots slightly more than the FP distribution. Nonetheless, these distributions and wave formations indicate that the region downstream of the jet was the same despite the differences in geometry.

Although the geometries were different, the $\lambda$ shock creating the attachment node upstream of the jet and the jet overexpansion downstream had the most significant impact on the surface pressures and JI force and moment. Integration of the surface pressures showed that the jet force and moment coefficients, $C_{Nji}$ and $C_{mji}$, were significantly different as shown in Table 5.9 which compares the force, moment and thrust coefficients as well as the amplification coefficients for the BOR and FP calculations.
Table 5.9 Coefficients For FP And BOR, Convergent Nozzle

<table>
<thead>
<tr>
<th>Body</th>
<th>$C_{Nji}$</th>
<th>$C_{mji}$</th>
<th>$C_T$</th>
<th>$\varepsilon_N$</th>
<th>$\varepsilon_m$</th>
</tr>
</thead>
<tbody>
<tr>
<td>FP</td>
<td>-3.95e-3</td>
<td>+3.87e-3</td>
<td>-1.83e-2</td>
<td>+1.22</td>
<td>+0.46</td>
</tr>
<tr>
<td>BOR</td>
<td>+0.38</td>
<td>+2.37</td>
<td>-2.75</td>
<td>+0.86</td>
<td>+0.76</td>
</tr>
</tbody>
</table>

The jet interaction force and moment coefficients and thrust coefficients for the BOR were significantly different because of the different ways that the areas were referenced between the flat plat and the BOR. The reference quantities used in these coefficients are shown in Table 5.10 with the FP quantities shown for reference.

Table 5.10 Reference Quantities For FP and BOR

<table>
<thead>
<tr>
<th>Body</th>
<th>$L_{ref}$ (ft)</th>
<th>$S_{ref}$ (ft$^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>FP</td>
<td>1.5</td>
<td>2.25</td>
</tr>
<tr>
<td>BOR</td>
<td>0.139</td>
<td>0.015</td>
</tr>
</tbody>
</table>

However, the amplification coefficients between the two bodies were comparable and Table 5.9 shows the jet force was attenuated in the BOR case while it was amplified in the FP case which can be attributed to the surface pressure perturbations wrapping around the side of the body. The jet moment however was attenuated in both cases with FP attenuated more than twice as much as the BOR. Further investigation into jet interaction forces and moments and their effect on BOR was done to examine the effect of a more efficient nozzle type, namely a convergent-divergent, or CD, nozzle. In the following section, the BOR with a CD nozzle at various PR are examined.
5.2.2 Convergent-Divergent (CD) Nozzle

5.2.2.1 Pressure Ratio Variation

In this final section, the BOR with a CD nozzle shown in Figure 5.153 operating at jet PR of 1000, 500 and 50 is compared to investigate the effect of the PR on the near field flow structure and jet interaction coefficients with a more efficient nozzle.

![Convergent-divergent nozzle geometry](image)

Figure 5.145 Convergent-divergent nozzle geometry

The nozzle inlet total pressures used for these calculations are shown in Table 5.11.

<table>
<thead>
<tr>
<th>Pressure Ratio (PR)</th>
<th>$P_{t_{jet}}$ (kPa)</th>
<th>$P_{t_{jet}}$ (psf)</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>491.6</td>
<td>10267.9</td>
</tr>
<tr>
<td>500</td>
<td>4916.3</td>
<td>102679.4</td>
</tr>
<tr>
<td>1000</td>
<td>9832.6</td>
<td>205358.8</td>
</tr>
</tbody>
</table>

Table 5.11 BOR Nozzle Inlet Total Pressures
Given these conditions, streamlines calculated through the nozzle at each PR are shown in Figure 5.146.

![Streamlines, CD nozzle, BOR, PR sweep](image)

Similar to the flat plate cases, the streamlines showed progressively increasing jet penetration and produced stronger shock and expansions with increasing PR. The upstream interactions for each PR are shown in Figure 5.147 through 5.149.
Figure 5.147 Upstream pressure contours and velocity vectors, BOR, PR=1000

Figure 5.148 Upstream pressure contours and velocity vectors, BOR, PR=500
Figure 5.149 Upstream pressure contours and velocity vectors, BOR, PR=50

The calculations show similar wave formations and flow structure as the BOR calculations with a convergent nozzle (Figure 5.137). As the PR decreased, the bow shock and barrel shock weakened and the interaction region shrunk while the recirculation and stagnation point moved downstream with a second recirculation emerging at PR 50. The extent of the influence of the upstream flow structure decreased with PR as shown in Figure 5.150.
Figure 5.150 shows the upstream surface pressure distributions for each PR examined. The character of these distributions is similar to the BOR calculations with a convergent nozzle indicating the separation topologies are similar except PR 50 which has the same character as the flat plate at PR above 100.

Separation topologies for each PR are shown in Figures 5.151 through 5.153.
Figure 5.151 Skin friction lines with pressure contours, BOR, CD nozzle, PR=50

Figure 5.152 Skin friction lines with pressure contours, BOR, CD nozzle, PR=500
Figures 5.151 through 5.153 show skin friction lines in black and surface pressure contours in white with the separation topology highlighted. As the PR increased from 50 to 500, a pitchfork bifurcation changed the phase portrait transforming two separation foci and one saddle point into a single separation node. There were two separation foci at PR 50. According to Tobak and Peake (1982), separation foci lead to horn vortices so it was assumed that horn vortices were present. However, the horn vortices were so small that the grid could not resolve them so it was assumed that they were insignificant. The horseshoe vortices on the other hand were significant.
Figure 5.154 Horseshoe vortices, BOR, CD nozzle, PR=50

Figure 5.155 Horseshoe vortices, BOR, CD nozzle, PR=500
Figures 5.154 through 5.156 show streamlines for each PR wrapped around the first pair of dividing surfaces forming horseshoe vortices around the jet. These horseshoe vortices were similar to the BOR with convergent nozzle horseshoe vortices with the vortices wrapping around the body perturbing the surfaces on the side of the body. Similar to the convergent nozzle calculations, a second pair of horseshoe vortices did not form because of the favorable pressure gradient created by the attachment node at the tip of the nose except at PR=50. At PR=50, a second pair of horseshoe vortices formed as the bow shock moved downstream with PR reduction. The pressure gradient between the attachment node near the jet exit and the jet intensified resulting in a second $\lambda$ structure, a second recirculation, an additional focus and saddle point with the flow and a second pair of horseshoe vortices. However, this pair of horseshoe vortices was small and, as in the FP cases, had a negligible influence on the surface pressures and jet interaction forces.
The same was true of the near field wake vortices which were ultimately eliminated with the bifurcation between PR=50 and 500 leaving the far field wake vortices and jet vortices with the horseshoe vortices as the prominent vortices in the near field, but as noted previously, the jet vortices had no identifiable effect on the surface pressures while the far field wake vortices shown in Figures 5.157 through 5.159 produced compression wavelets having a direct impact on the surface pressure and JI force and moment.

Figure 5.157 Far field wake vortices, BOR, CD nozzle, PR=50
Figures 5.157 through 5.159 show streamlines wrapping around the dividing surface emanating from the downstream global separation lines with skin friction lines.
shown in black and the surface pressure contours shown in white. The growth of the far field wake vortices created compression waves resulting in a gradual pressure rise in the downstream pressure distributions shown in Figure 5.160.

Figure 5.160 shows the downstream surface pressure distributions for each PR examined which all have the same character as the BOR convergent nozzle shown in Figure 5.105. The general character includes a massive overexpansion followed by a gradual recompression with larger overshoot as PR increased. As noted previously, the overshoot was a result of the shock emanating from the Mach disk reflecting off the body, yet the greatest impact on the surface pressures and JI force and moment came from the jet overexpansion. The jet overexpansion and upstream $\lambda$ shock provided the largest perturbation to the near field surface pressures. Integration of these surface pressures showed the JI force and moment, $C_{Nij}$ and $C_{mij}$, increased dramatically with PR.
as did the thrust coefficient, $C_T$. Table 5.12 summarizes the performance coefficients for the BOR with a CD nozzle for each PR examined.

<table>
<thead>
<tr>
<th>PR</th>
<th>$C_{Nij}$</th>
<th>$C_{mij}$</th>
<th>$C_T$</th>
<th>$\varepsilon_N$</th>
<th>$\varepsilon_m$</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>+0.14</td>
<td>+0.92</td>
<td>-0.31</td>
<td>+0.56</td>
<td>+0.17</td>
</tr>
<tr>
<td>500</td>
<td>+0.61</td>
<td>+3.32</td>
<td>-3.24</td>
<td>+0.81</td>
<td>+0.71</td>
</tr>
<tr>
<td>1000</td>
<td>+1.05</td>
<td>+5.45</td>
<td>-6.50</td>
<td>+0.84</td>
<td>+0.76</td>
</tr>
</tbody>
</table>

The increase in thrust coefficient, $C_T$, was directly proportional to the increase in PR while the increase in $C_{Nij}$ and $C_{mij}$ were nearly proportional. The amplification coefficients on the other hand were not with a significant increase occurring with the pitchfork bifurcation between PR 50 and 500 following by nearly constant coefficients as PR increased to 1000.

Comparing PR=500 and 1000 with the calculations of the convergent nozzle at PR=2000, it shows the amplification coefficients are nearly constant even with very different thrust coefficients. From a broad view, the flow structure and separation topologies were very similar between these calculations indicating for a BOR, the nozzle type had little effect on the amplification coefficients and after the phase portrait reached a constant pattern, neither does PR.
CHAPTER 6

CONCLUSIONS AND RECOMMENDATIONS

The interaction between a transverse jet and freestream resulted in considerable surface pressure disturbances as well as significant modifications to the freestream in the near field of the jet. This investigation showed that amendments to the currently accepted near-field flow structure model are required by including a pair of near-field and far-field wake vortices, a series of compression waves and a downstream shock reflection. Moreover, phase portrait bifurcations substantially altered the flow structure making it impractical to identify a single flow structure model across all flow conditions. Table 6.1 summarizes the singularities present at the conditions examined in this study.
Table 6.1 Singularity Summary

<table>
<thead>
<tr>
<th>Body Type</th>
<th>Boundary Layer Type</th>
<th>Nozzle Type</th>
<th>Mach</th>
<th>Pressure Ratio</th>
<th>Saddle Points</th>
<th>Nodes</th>
<th>Foci</th>
</tr>
</thead>
<tbody>
<tr>
<td>FP</td>
<td>Laminar</td>
<td>C</td>
<td>0.3</td>
<td>1.5</td>
<td>3 ½</td>
<td>1 ½</td>
<td>1</td>
</tr>
<tr>
<td>FP</td>
<td>Turbulent</td>
<td>C</td>
<td>0.3</td>
<td>1.5</td>
<td>2</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>FP</td>
<td>Turbulent</td>
<td>C</td>
<td>0.3</td>
<td>5</td>
<td>3</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>FP</td>
<td>Turbulent</td>
<td>C</td>
<td>2.0</td>
<td>5</td>
<td>4</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>FP</td>
<td>Turbulent</td>
<td>C</td>
<td>2.0</td>
<td>15</td>
<td>5</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>FP</td>
<td>Turbulent</td>
<td>C</td>
<td>2.0</td>
<td>100</td>
<td>4</td>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>FP</td>
<td>Turbulent</td>
<td>C</td>
<td>2.0</td>
<td>500</td>
<td>4</td>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>FP</td>
<td>Turbulent</td>
<td>C</td>
<td>2.0</td>
<td>1000</td>
<td>4</td>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>FP</td>
<td>Turbulent</td>
<td>C</td>
<td>2.0</td>
<td>50</td>
<td>4</td>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>FP</td>
<td>Turbulent</td>
<td>C</td>
<td>2.5</td>
<td>2000</td>
<td>4</td>
<td>3</td>
<td>0</td>
</tr>
<tr>
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<td>Turbulent</td>
<td>C</td>
<td>3.0</td>
<td>2000</td>
<td>4</td>
<td>3</td>
<td>0</td>
</tr>
<tr>
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<td>C</td>
<td>3.5</td>
<td>2000</td>
<td>4</td>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>FP</td>
<td>Turbulent</td>
<td>C</td>
<td>4.0</td>
<td>2000</td>
<td>4</td>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>BOR</td>
<td>Turbulent</td>
<td>C</td>
<td>2.0</td>
<td>2000</td>
<td>4</td>
<td>6</td>
<td>0</td>
</tr>
<tr>
<td>BOR</td>
<td>Turbulent</td>
<td>CD</td>
<td>2.0</td>
<td>1000</td>
<td>5</td>
<td>5</td>
<td>2</td>
</tr>
<tr>
<td>BOR</td>
<td>Turbulent</td>
<td>CD</td>
<td>2.0</td>
<td>500</td>
<td>4</td>
<td>6</td>
<td>0</td>
</tr>
<tr>
<td>BOR</td>
<td>Turbulent</td>
<td>CD</td>
<td>2.0</td>
<td>50</td>
<td>4</td>
<td>6</td>
<td>0</td>
</tr>
</tbody>
</table>

In subsonic freestreams, the influence of the state of the boundary layer and jet pressure ratio were investigated. Both bifurcated the phase portrait, but the state of the boundary layer had no effect on the amplification coefficient while the jet pressure ratio had only a modest effect on the moment amplification coefficient. As the phase portraits bifurcated because of the change in the state of the boundary layer, the separation zone disappeared with the horseshoe, near field and far field wake vortices while the separation foci and horn vortices remained.

When the freestream is supersonic, the phase portrait bifurcated again producing a massive separation zone extending far downstream and radically altering the force amplification coefficient from complete nullification to more than 150% amplification.
The massive separation zone was common at all supersonic conditions, but as the jet pressure ratio was increased, the phase portrait bifurcated until approximately PR=100 before stabilizing while the amplification coefficients decreased with the moment amplification coefficient dropping below 1.0. As Mach number increased, the phase portrait and near field flow structure remained unchanged while the amplification coefficients increased to near 3. Reasonable correlation of the amplification coefficients for flat plates over a wide range of jet pressure ratios and supersonic freestream Mach numbers was achieved by defining a modified momentum flux ratio while recognizing further research at transonic Mach numbers would be required to examine the validity of this parameter over an entire flight regime.

The phase portrait of a transverse jet issuing from a body of revolution into a supersonic freestream was very similar to the flat plate counterpart with the massive separation wrapping around the body and extending far downstream. However, the additional attachment node at the nose tip eliminated the second pair of horseshoe vortices affecting the surface pressures and reducing the amplification coefficients below 1.0.

Recommendations for further research include surface oil and particle image velocimetry test to verify the horn, near- and far-field wake vortices, examination of the effects of angle of attack, both negative and positive, on jet interaction, phase portrait and flow structure and investigation of the BOR at various freestream Mach numbers, both supersonic and transonic. In addition, the washout component of JI has been completely ignored in this study. Very little literature is available on the effect of the various sets of
vortices on downstream control surfaces so a meaningful examination of the effect of these flow structure on downstream flow fields around wings and tail fins would be very productive.
APPENDIX A

RESIDUAL HISTORIES
Figure A.1 Residual histories, $M=0.3$, no jet, FP

Figure A.2 Residual histories, $M=0.3$, various jets, FP
Figure A.3 Residual histories, $M=2$, FP

Figure A.4 Residual histories, no jets, FP
Figure A.5 Residual histories, PR=2000, FP

Figure A.6 Residual histories, $M=2$, BOR
APPENDIX B

FORCE HISTORIES
Figure B.1 Force Histories, \( M=0.3 \), no jet, FP

Figure B.2 Force histories, \( M=0.3 \), various jets, FP
Figure B.3 Force histories, $M=2$, FP

Figure B.4 Force histories, no jets, FP
Figure B.5 Force histories, PR=2000, FP

Figure B.6 Force histories, $M=2$, BOR
REFERENCES


BIOGRAPHICAL INFORMATION

Dean Anthony Dickmann was born to John and Ann Dickmann on June 9th, 1961 in Alton, Illinois. After graduating from Poland High School in Poland, Ohio, he was admitted to the Ohio State University for study in the mechanical engineering department. He began his professional career as a cooperative education employee at Jones and Laughlin Corporation, Seamless Pipe Division in Campbell, Ohio. After two semesters as a co-op student, he graduated with a BSME and took a position at Goodyear Aerospace Corporation in Akron, Ohio as a design engineer servicing the Vertical Launch Anti-Submarine Rocket (VLA) program for the U.S. Navy. During his tenure at Goodyear Aerospace, he began pursuing a master’s degree in fluid and thermal science at Case Western Reserve University (CWRU) in Cleveland, Ohio under the supervision of Dr. Eli Reshotko. Before graduation from CWRU, he took a post at General Dynamics Corporation in Fort Worth, Texas as a propulsion analyst servicing several projects in Mach 4-6 propulsion systems, advanced aircraft inlet design and F-16 signature suppression nozzle designs while finishing his master’s thesis entitled “A Computational Study of the Cooling Characteristics of Axisymmetric Supersonic Ejector Nozzles“ in 1992.

In 1996, he took a position at Texas Instruments (TI), Defense Division, in Lewisville, Texas as an aero-thermal analyst servicing programs such as JSOW, JASSM, ERGM and XM-982 and began work on this doctoral degree. During this period of
study, TI sold their defense division to Raytheon and closed the Lewisville facility. Dean took a position in the wafer processing industry with Millipore Corporation as a consulting engineer working the development of a sub-ambient mass flow controller until a position opened in the aerodynamics group at Lockheed Martin Missiles and Fire Control (LMMFC) in Grand Prairie, Texas in 2001 where he continues to hold the title of senior staff engineer.