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Preliminary design of the MHD accelerator for an arc-heated, MHD-augmented hypersonic wind tunnel

Liu, Cheng-Jin, M.S.A.E.
The University of Texas at Arlington, 1990
PRELIMINARY DESIGN OF THE MHD ACCELERATOR
FOR AN ARC-HEATED, MHD-AUGMENTED
HYPERSONIC WIND TUNNEL

The members of committee approve the masters thesis
of Cheng-Jin Liu

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Supervising Professor  

Donald D. Seath

Frank K. Lu
PRELIMINARY DESIGN OF THE MHD ACCELERATOR
FOR AN ARC-HEATED, MHD-AUGMENTED
HYPERSOニック WIND TUNNEL

by

CHENG-JIN LIU

Presented to the Faculty of the Graduate School of
The University of Texas at Arlington in Partial Fulfillment
of the Requirements
for the Degree of

MASTER OF SCIENCE IN AEROSPACE ENGINEERING

THE UNIVERSITY OF TEXAS AT ARLINGTON
May 1990
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Moreover, I would like to thank my parents, relatives, and friends for all their love and support during my study in the United States.

January 20, 1990
ABSTRACT

PRELIMINARY DESIGN OF THE MHD ACCELERATOR
FOR AN ARC-HEATED, MHD-AUGMENTED
HYPERSONIC WIND TUNNEL

Publication No.________

Cheng-Jin Liu, M.S.
The University of Texas at Arlington, 1990

Supervising Professor: Donald R. Wilson

In order to reach higher Mach numbers in the test section of hypersonic arc-heated wind tunnels, without incurring non-equilibrium flow in the nozzle expansion, the concept of magnetohydrodynamic (MHD) augmentation is proposed. This thesis is to assess the feasibility of this concept, and to estimate the performance enhancement possible by applying MHD augmentation to an arc-heated hypersonic wind tunnel currently being developed at the University of Texas at Arlington.

Two major operating modes of the MHD accelerator are analyzed, the linear Faraday accelerator and Hall accelerator. For the former, a significant gain in simulation capability can be obtained for a MHD accelerator operated under the constraints of constant
temperature and cross sectional area. The Hall configuration is shown to be not feasible.
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<tr>
<td>A</td>
<td>area of cross section</td>
<td>m²</td>
</tr>
<tr>
<td>Ai</td>
<td>MHD accelerator inlet area (at x=0 m)</td>
<td>m²</td>
</tr>
<tr>
<td>Ae</td>
<td>MHD accelerator exit area (at x=1 m)</td>
<td>m²</td>
</tr>
<tr>
<td>a</td>
<td>speed of sound</td>
<td>m/s</td>
</tr>
<tr>
<td>B</td>
<td>magnetic flux intensity</td>
<td>T</td>
</tr>
<tr>
<td>Bz</td>
<td>z-component of magnetic flux intensity</td>
<td>T</td>
</tr>
<tr>
<td>cp</td>
<td>specific heat at constant pressure</td>
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<tr>
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<tr>
<td>h</td>
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<tr>
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<tr>
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<td>W/m³</td>
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(Greek)

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<td>$\gamma$</td>
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<td>$\mu$</td>
<td>viscosity coefficient</td>
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<tr>
<td>$\mu$</td>
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<td></td>
<td>($= \text{V s/(A m)}$)</td>
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<tr>
<td>$\rho$</td>
<td>density</td>
<td>kg/m³</td>
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<tr>
<td>$\sigma$</td>
<td>electrical conductivity</td>
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</tr>
<tr>
<td></td>
<td>($= \text{mho/m}$)</td>
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\( \eta \) local electrical efficiency

(Subscripts)

c \hspace{1cm} \text{core}

e \hspace{1cm} \text{MHD accelerator exit condition (at } x = 1 \text{ m)}
i \hspace{1cm} \text{MHD accelerator inlet condition (at } x = 0 \text{ m)}
o \hspace{1cm} \text{reference condition}
t \hspace{1cm} \text{total (stagnation) condition}
x, y, z \hspace{1cm} \text{Cartesian coordinate components}
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CHAPTER 1
INTRODUCTION

1.1 General Concept

Conventional arc-heated wind tunnels are often used for supersonic and hypersonic continuous (in seconds) flow experiments. Nevertheless, it is difficult to reach high Mach numbers, without incurring departures from vibrational and chemical equilibrium in the nozzle flow expansion. The introduction of non-equilibrium flow in the nozzle expansion results in a test section flow that does not properly simulate the chemical composition of the atmosphere, which introduces a large uncertainty in interpretation of subsequent real-gas effects that occur in the flow field about models placed in the wind tunnel. The magnetohydrodynamic (MHD) augmented arc-heated wind tunnel was originally proposed to resolve this problem.

The general features of the proposed facility are shown in Fig. 1.1 (Ref. 1). Air would be initially preheated, seeded and ionized by a conventional arc-heater, then passed through a low Mach number supersonic nozzle into the accelerating channel where the electrical energy would be added to the plasma in the form of directed kinetic energy as well as thermal energy. A post-nozzle would be placed at the accelerator exit to further expand the flow to a hypersonic Mach number. The flow then passes through the test section and into a
diffuser for partial pressure recovery prior to entering the vacuum tank.

The primary advantages of this concept are the reduction of the initial pressure and temperature levels that must be achieved in the arc heater in order to subsequently expand the flow to high velocities, and the possibility of achieving equilibrium flow conditions in the test section.

1.2 Design Requirements and Criteria

In order to maintain equilibrium, or at least near-equilibrium, flow in the test section, the temperature throughout the accelerator should be maintained at or below 2500 K, which is about the temperature that oxygen begins to dissociate. Nevertheless, it is also necessary to maintain a sufficiently high temperature to allow the plasma to have adequate electrical conductivity in order to efficiently operate the MHD accelerator. The electrical conductivity would decrease much more rapidly as the temperature drops if the temperature is less than 3000 K (see section 3.2). So the temperature is fixed at 2500 K in most of the cases discussed in this paper.

For a subsonic flow, although the MHD accelerator exerts a positive body force on the fluid (i.e., the MHD body force and the fluid flow have the same direction), the flow velocity still could decelerate. For an isothermal constant-area duct with subsonic flow, a positive body force together with thermal energy transfer
out of the flow would also decelerate the flow speed (section 5-7 of Ref. 2). It is very difficult to accelerate the flow directly from subsonic to supersonic speeds by application of the body force with the constraint of specific temperature and area distributions. For this reason, the flow is first expanded to a low supersonic Mach number by means of a conventional converging-diverging nozzle prior to entering the MHD accelerator.

Considerations regarding nozzle cooling requirements and the necessity of adequate entry length for build-up of the electromagnetic fields led to the selection of an accelerator entrance Mach number of 1.8 for all of the design cases considered. Certainly, the inlet position and Mach number are variable, depending on the particular limitations of the facilities. There is a restriction for the exit Mach number of the accelerator also. The Mach number would be too high to have efficient energy conversion if it is greater than about six. The best Mach number for the exit is about four to six (Ref. 3).

The longer the accelerator channel length, the more severe the boundary layer effects would be. These include surface friction, flow non-uniformity, flow separation, voltage drop, electrical shorting, and a large thermal energy loss which is significant for a high temperature facility. However, the channel length to height, or width, ratio can not be less than about twenty to allow for sufficient electromagnetic energy addition, so the flow can be approximated as a quasi-one-dimensional flow without end effects if the area variation is small. The mass flow rate was given as
0.093 kg/s, and the static pressure at the accelerator inlet is fixed at 50,600 Pa (half atmosphere). These values were chosen to match the existing UTA supersonic arc-heated wind tunnel specifications. Together with the other design criteria, $T=2500$ K and $M_i=1.8$, the required inlet area can be calculated as $A_i=0.00075$ m$^2$. Thus both the width and height of the square cross-sectional geometry, which is the only geometry considered, are $H=W=0.0274$ m. The accelerator channel length considered in this paper would not be more than 1 m.

The NASA/UTA MHD79 computer code (Ref. 4) is the major computational tool used for the design study. Only the quasi-one-dimensional calorically perfect gas (CPG) model with $\gamma=1.33$ was applied. Because we are only interested in the high temperature isothermal ($T=2500$ K) core flow of air, the assumption of a CPG is suitable. The two most common designs of MHD accelerators, the linear Faraday accelerator and the Hall accelerator (Ref. 4), are considered.
CHAPTER 2

ANALYTICAL MODEL

2.1 Definition of an Accelerator

In this paper, the MHD accelerator is defined as a device in which the electric power input will exert a positive (i.e. the same direction as the flow velocity) MHD body force on a fluid. In other words, the accelerator does positive work on the fluid, as well as adding thermal energy which is transformed from electrical energy. As mentioned before, it is not always guaranteed that the flow velocity would be increased in an accelerator although it does in most supersonic cases.

In contrast to the accelerator, an MHD generator is a device in which a negative MHD body force is exerted on the fluid, and electrical power output is generated. In the generator, a negative flow work is done on the fluid that causes electrical energy to be generated, just like in a conventional power plant. The special case in which a negative MHD body force is applied to the fluid, even though the input electric power is still positive is referred to as a "decelerator." In general, the same channel can be operated either as an accelerator, generator, or a decelerator, and is referred to as an MHD converter.
2.2 Coordinates and Units

The right-handed Cartesian coordinates are used throughout this paper. The flow direction (u direction) is assigned to be the positive \( x \)-direction (Fig. 2.1). The magnetic flux intensity \( B \) is assigned to be in the positive \( z \)-direction, which is also assumed to be constant everywhere, so \( B=Bz= \) constant. The insulated electrodes are placed on the walls perpendicular to the \( y \)-axis. Both the electric field and the flow velocity are perpendicular to the magnetic field because the \( E \)-field is in the positive \( y \)-direction.

Only the International System of Units (SI Units) is used in this paper (Ref. 5). The units are listed with the symbols in the Symbols and Units section in front of the text, where the unit can be found if it is not noted or specified in the paper.

2.3 Linear Faraday and Hall Converters

A linear Faraday converter is shown in Figs. 2.1 and 2.2a. The key points of the Faraday converter channel are that the electrodes are segmented along the \( x \)-direction and insulated, and each pair of opposing electrodes at the same axial position are connected to either a load resistance in a generator, or a power supply in a accelerator. Because of the segmented electrodes, \( Jx=0 \) is the basic assumption for Faraday MHD converters.

As shown in Fig. 2.2b, for a Hall converter, only one load resistance or power supply is connected to the first and last pairs of electrodes instead of each pair of electrodes being connected to
an independent load or power supply as in a Faraday converter. Moreover, each opposing pair of anodes and cathodes is short-circuited in a Hall converter. Thus ideally, $E_Y = 0$, because of the short-circuited electrodes.

2.4 Core Flow Equations

The quasi-one-dimensional MHD channel core flow equations (Ref. 3) are given by

(2.4.1) \[ \rho \ u \frac{du}{dx} = -\frac{dp}{dx} + J_y B_z \]

(2.4.2) \[ \rho \ u \frac{dH_t}{dx} = \rho \ u \left( \frac{dh}{dx} + u \frac{du}{dx} \right) = J_x E_x + J_y E_y \]

(2.4.3) \[ \frac{dm}{dt} = \rho \ u A \]

(2.4.4) \[ \rho = \rho(p, T) \]

These equations are the momentum, energy, continuity, and state equations respectively, and are based on the following assumptions (Ref. 6):

a. quasi-one-dimensional flow; i.e., transverse variations of any fluid or electromagnetic property are neglected.

b. steady flow

c. inviscid flow

d. no heat transfer to the channel walls

e. thermodynamic and chemical equilibrium
f. MHD approximations are valid

g. the magnetic Reynolds number ($= \sigma \mu uL$) is small compared to unity

h. the field orientation is as discussed in Section 2.2

These equations can be transformed to a more convenient form for numerical integration by solving for explicit relations for two independent core flow property derivatives (Ref. 4). For example, these equations can be solved explicitly for $dp/dx$ and $dT/dx$ as

\begin{equation}
\frac{dp}{dx} = - \frac{u^2 \left( \frac{\partial p}{\partial T} \right)_p \left[ Jx \, Ex + Jy \left( Ey - u \, Bz \right) \right]}{\rho \, u \, c_p \left( M^2 - 1 \right)} - \frac{\rho \, u \, c_p \left( Jy \, Bz + \frac{\rho \, u^2}{A} \frac{dA}{dx} \right)}{\rho \, u \, c_p \left( M^2 - 1 \right)}
\end{equation}

\begin{equation}
\frac{dT}{dx} = \left[ u^2 \left( \frac{\partial p}{\partial T} \right)_T - 1 \right] \frac{\left[ Jx \, Ex + Jy \left( Ey - u \, Bz \right) \right]}{\rho \, u \, c_p \left( M^2 - 1 \right)} + \frac{T \left( \frac{\partial p}{\partial T} \right)_p \left( Jy \, Bz + \frac{\rho \, u^3}{A} \frac{dA}{dx} \right)}{\sigma \, u \, c_p \left( M^2 - 1 \right)}
\end{equation}

If we assume that expressions for the thermodynamic properties, transport properties and the electromagnetic field, which will be developed in the following sections, are available,
then \( p(x+\Delta x) \) and \( T(x+\Delta x) \) can be easily solved by numerical integration once \( dA/dx \) or \( A(x) \) is specified. Then together with the specified value of area at \( x+\Delta x \), all of the remaining flow properties at this position can be obtained. Note that in Eqs. (2.4.5) and (2.4.6), the three variables \( dp/dx \), \( dT/dx \) and \( dA/dx \) are mutually linear. Therefore, it is also possible to express both \( dp/dx \) and \( dA/dx \), or \( dT/dx \) and \( dA/dx \), as functions of specified variations of either \( dT/dx \) or \( dp/dx \). Thus, all flow properties can also be calculated if either the temperature or pressure distribution is specified. On the other hand, if the core flow velocity distribution \( u(x) \) is regarded as a specified design variable, Eqs. (2.4.1)-(2.4.4) can be solved as (Ref. 4):

\[
(2.4.7) \quad dp/dx = \rho u \frac{du}{dx} + Jy Bz \\
(2.4.8) \quad dT/dx = \left[ u^2 T \left( \frac{\partial \rho}{\partial T} \rho \right) \frac{du}{dx} - \left( \frac{T}{\rho} \right) \left( \frac{\partial \rho}{\partial T} \right) \rho Jy \right. \\
\left. + Jx Ex + Jy (Ey - u Bz) \right] / (\rho u cp)
\]

For this case, the remaining flow properties are obtained after the pressure and temperature variations are calculated by numerical integration.

One of five specific flow properties can be chosen as the design option to be specified; these are core flow area \( A \), pressure \( p \), temperature \( T \), velocity \( u \), or Mach number parameter \( \gamma M^2 \). In other words, if one of the five specific flow properties is specified, all the other flow properties can be calculated. Eqs. (4.4.5)-(4.4.8) give the core flow differential equations for the first four, \( A \), \( p \), \( T \) and \( u \),
design options. For the $\gamma M^2$ specified option, similar equations can be found in Ref. 4.

2.5 Electromagnetic Field Relations

In order to solve the core flow equations, the electromagnetic variables, such as $B_z$, $J_y$, $J_x$, $E_y$ and $E_x$, must be determined first. The magnetic field intensity $B_z$ is generated by an external electromagnet, and is thus regarded as a known specification. The magnetic field intensity is also assumed to be uniform and constant all over the MHD channel.

Ohm's Law with diffusion and electron inertia neglected (Ref. 3) is

\[(2.5.1) \quad J = \sigma (E + u \times B) - \beta J \times B / B\]

Written in component form for the design model which we are dealing with, Ohm's law becomes

\[(2.5.2) \quad J_x = \sigma E_x - \beta J_y\]
\[(2.5.3) \quad J_y = \sigma (E_y - u B_z) + \beta J_x\]

Any two variables of $J_x$, $J_y$, $E_x$ and $E_y$ can be solved by the two simultaneous equations (2.5.2) and (2.5.3) if the other two are known. For example, the current densities can be solved as

\[(2.5.4) \quad J_x = \sigma [ E_x - \beta (E_y - u B_z) ] / (1 + \beta^2)\]
\[(2.5.5) \quad J_y = \sigma [ (E_y - u B_z) + \beta E_x ] / (1 + \beta^2)\]
For a Faraday MHD converter, Jx=0 (section 2.3), and Ey is specified by the input power supply voltage, thus Jy and Ex will be given by

(2.5.6) \[ Jy = \sigma (Ey - u Bz) \]
(2.5.7) \[ Ex = \beta (Ey - u Bz) \]

For a Hall MHD converter, Ey=0, and if Jx is specified, Jy and Ex would be given by

(2.5.8) \[ Jy = \sigma ( -u Bz + \beta Ex ) / (1 + \beta^2) \]
(2.5.9) \[ Ex = Jx [ (1 + \beta^2)/\sigma ] - \beta u Bz \]

2.6 Thermodynamic Properties

A calorically perfect gas model is applied to the design calculations. With this assumption, the equation of state is

(2.6.1) \[ \rho = p / \left[ (R/M) T \right] = p / (RT) \]

where the universal gas constant \( R = 8314.34 \) J/(kmol K), and the molecular weight \( M = 28.97 \) kg/kmol for air. The partial derivatives appearing in Eqs. (2.4.5)-(2.4.8) become

(2.6.2) \[ (\partial p / \partial T)_p = \partial (p/RT)_p / \partial T = - p/(RT^2) \]
(2.6.3) \[ (\partial p / \partial p)_T = \partial (p/RT)_T / \partial p = 1/(RT) \]

and the specific heat can be calculated from

(2.6.4) \[ c_p = \gamma R / (\gamma - 1) \]
The stagnation conditions for pressure and enthalpy will be calculated from the following equations once the static properties are calculated from the solution of the core flow equations (section 2.4).

\[ P_t = p \left[ 1 + (\gamma - 1)M^2/2 \right]^{\gamma/(\gamma-1)} \]

\[ H_t = h + u^2/2 = cp T + u^2/2 \]

The specific heat ratio (isentropic exponent) is assumed to be \( \gamma = 1.33 \).

We have to keep in mind that the stagnation conditions calculated by Eqs. (2.6.5) and (2.6.6) are not accurate for high-temperature gas flows, but represent the condition that would be obtained via a frozen isentropic compression process from static to stagnation state because of the calorically perfect gas assumption. However, they are still adequate reference conditions for understanding the flow characteristics.

### 2.7 Transport Properties

The electrical transport properties are obtained from empirical curve fit relations (Ref. 6),

\[ \sigma/\sigma_0 = (T/T_o)^{0.75} \left( p_o/p \right)^{0.5} \exp \left[ \text{Ei}(1 - T_o/T)/(2k T_o) \right] \]

\[ \beta/\beta_0 = (B/B_o) \left( p_o/p \right) \left( T/T_o \right)^{0.5} \]
The reference values of $\sigma_o$, $\beta_o$, $B_o$, $T_o$ and $p_o$ are selected to approximately coincide with the mean values for the estimated temperature and pressure range.

2.8 Performance Parameters

The integration over the volume of the channel of a number of important performance parameters is performed here. The integrated (overall) power input for the Faraday and Hall converters respectively are

\begin{align*}
\text{(2.8.1)} \quad P \text{ (Faraday)} &= -\int_0^x V_y J_y W \, dx \\
&= \int_0^x E_y J_y W \, H \, dx
\end{align*}

\begin{align*}
\text{(2.8.2)} \quad P \text{ (Hall)} &= I_x \int_0^x E_x \, dx \\
&= \int_0^x J_x E_x W \, H \, dx
\end{align*}

where $V_y = -\int_0^H E_y \, dy$ and $I_x = J_x W \, H$

The local input electric power density is given by

\begin{align*}
\text{(2.8.3)} \quad P_d = J \cdot E = J_x E_x + J_y E_y
\end{align*}

Hence, the power density for a Faraday converter is
(2.8.3a) \[ P_d = J_y \, E_y = J_y (\frac{J_y}{\sigma} + u \, B_z) \]
\[ = \sigma^{-1} \, J_y^2 + u \, B_z \, J_y \] (Faraday converter)

and for a Hall converter is

(2.8.3b) \[ P_d = J_x \, E_x = \sigma^{-1} (1 + \beta^2) \, J_x^2 - \beta \, u \, B_z \, J_x \] (Hall converter)

If the value of power or power density becomes negative, this implies that the electric energy is being extracted from the fluid (power generator). In other words, the electrical energy is being generated by the converter instead of being consumed.

Another important local performance parameter to be used in the analysis work later is the local electrical efficiency, \( \eta \), which is defined as the ratio of flow work density (local flow work done by the body force, in Joule per cubic meter per second) to input power density for an accelerator.

(2.8.4) \[ \eta = u \cdot (J \times B) / (J \cdot E) \] (accelerator)

Eq. (2.8.4) reduces to

(2.8.5) \[ \eta = \frac{J_y \, u \, B_z / (J_y \, E_y)}{u \, B_z / E_y} = \frac{B_z / E_y}{K^{-1}} \] (Faraday accelerator)

for a Faraday accelerator, where \( K = E_y / (u \, B_z) \) is the Faraday load factor. The corresponding relation for a Hall accelerator is

(2.8.6) \[ \eta = \frac{J_y \, u \, B_z}{(J_x \, E_x)} \] (Hall accelerator)
The local electric efficiency is defined in an opposite way for a generator as the ratio of power density which is negative to flow work density done by the fluid, i.e., the ratio of useful electric power density output to the mechanical work density done by the gas in pushing itself through the magnetic field. That is

\[(2.8.7) \quad \eta = \frac{J \cdot E}{u \cdot J \times B} \quad \text{(generator)}\]

Thus for a Faraday generator

\[(2.8.8) \quad \eta = \frac{Jy \cdot Ey}{(Jy \cdot u \cdot Bz)} = \frac{Ey}{(u \cdot Bz)} = K \quad \text{(Faraday generator)}\]

The value of \( \eta \) is always between zero and unity for either an accelerator or generator. The difference between numerator and denominator of \( \eta \) is the "Joule dissipation," which is nothing but thermal energy translated from part of the input electrical energy for an accelerator, or part of the consumed kinetic energy for a generator.

The local electromagnetic performance will be discussed in the next section.

2.9 Local Electromagnetic Performance

For a Faraday converter, when \( K = \frac{Ey}{(u \cdot Bz)} > 1 \) or \( Ey > u \cdot Bz \), we have

\[(2.9.1) \quad Jy = \sigma (Ey - u \cdot Bz) > 0\]
\[(2.9.2) \quad Pd = Jy \cdot Ey > 0\]
and the body force exerted on the fluid per unit volume is \( J \times B \) which is positive also. That is, when \( K > 1 \), the fluid is accelerated by a positive body force as a result of the positive input power. Therefore, at least at this local point, this converter behaves as an accelerator (Fig. 2.3).

If \( 0 < K < 1 \), we have \( J_y < 0 \) and \( P_d = J_y E_y < 0 \), because \( E_y = K u B_z > 0 \) with fixed \( B \)-field. So for this case both the body force \( J \times B \) and the power density are negative. The converter thus behaves as a generator now.

Finally, if \( K < 0 \), both \( E_y \) and \( J_y \) would be negative, but \( P_d = J_y E_y \) would become positive. So with a negative body force and positive input power, the converter behaves as a decelerator.

For the intermediate point, the Faraday converter corresponding to \( K = 1 \) where we have \( J_y = P_d = 0 \), although the electric efficiency \( \eta \) is maximum, the converter is just an ordinary channel without MHD energy conversion. All the above cases are shown in Fig. 2.3 with the corresponding values of electric efficiency.

A similar analysis can be performed for a Hall converter. The results are shown in Fig. 2.4. Note that, supposing all the other parameters are fixed, as \( J_x \) is decreased from a positive value, the operation of the converter would transfer from an accelerator to a short period as a decelerator, then to a generator, and finally to a decelerator again but with power output when \( J_x \) becomes negative.

In the next chapter, we will go through the different accelerator cases one by one with the computational results.
CHAPTER 3
RESULTS AND DISCUSSIONS

The Faraday accelerator cases are discussed first. These include cases of temperature and load factors specified (fixed), area and load factors specified, both temperature and area fixed, and fixed temperature with specified increasing area. The computational results for Hall accelerators are not so promising. They will be shown later.

3.1 Faraday Accelerators with Fixed Temperature and Load Factors

Figs. 3.1.1-Fig. 3.1.4 are the distributions of flow properties along the channel for load factors $K = 1.1, 1.15, 1.2$ and $1.3$ with fixed temperature $T = 2500 \, \text{K}$ in all cases. The axial variation of the properties shown in Fig. 3.1.1 are gradual and nearly linear. All of the properties increase except the area and static pressure. However, the problem with this design case is that insufficient power is added at the low value of load factor to reach high Mach number.

For the case of $K = 1.15$ (Fig. 3.1.2), flow properties vary more rapidly than the former one, but the exit Mach number is still less than 3.5. The cross-sectional area $A$ decreases at first, then
increases. For $K = 1.2$ (Fig. 3.1.3), it is encouraging that a Mach number greater than four is reached at the position of $x = 0.75$ m, but at the same position, the area is almost divergent and the computation is terminated by the error of negative pressure appearing. The situation is even worse for the case of $K = 1.3$ (Fig. 3.1.4). The computation is terminated for the same reason before $x = 0.4$ m, where the area is severely divergent also.

A significant area variation would cause a significant problem for MHD channel design. Shock waves may be generated any place where the area is changed. Although a properly designed post nozzle can compensate for the expansion wave generated by the divergent exit of the MHD channel, it is still difficult to manufacture practical electrode geometries to fit a large area variation. Also, the quasione-dimensional flow assumption will not be valid. In conclusion, the cases shown in Figs. 1.1.3 and 1.1.4 may be acceptable from a performance standpoint, but are not good designs.

As mentioned before, no MHD body force would be exerted on the fluid if $K = 1$. Together with the restraint of $T = \text{constant}$, every flow property will be constant like an ordinary constant area duct when $K$ equals to unity. So for the cases corresponding to $K$ less than 1.1, the results will look like those shown in Fig. 3.1.1 but every curve would be more horizontal. On the other hand, for $K$ greater than 1.3, the rates of power addition and flow property variations are too great. For example, with $K = 1.5$, the computation is terminated before $x = 0.15$ m.
The transport properties $\sigma$ and $\beta$ are functions of both temperature and pressure (section 2.7). The relations between $\sigma$ and pressure, and $\beta$ and pressure are shown in Fig. 3.1.5 for the data adopted from the solution of the case for $K = 1.3$.

An interesting chart is shown in Fig. 3.1.6 which is a comparison between the cases shown in Figs. 3.1.1 to 3.1.4. The lines of input power are almost coincident, so the same amount of electrical energy is required to reach the same Mach number (but the case of lower load factor requires a longer channel length). Nevertheless, the static pressure lines are significantly divergent. In the hypersonic wind tunnel design criteria, both Mach number and static pressure at the accelerator exit are important, because in the post-nozzle, a higher inlet static pressure can drive the fluid to higher Mach numbers for the same exhaust pressure. In Fig. 3.1.6, we can obviously conclude that the higher load factor results in lower static pressure at the same Mach number. The reason is that a higher load factor is just the same as a lower electric efficiency, $\eta = K^{-1}$ (section 2.8), which will correspond to the more input energy transferred to thermal energy instead of kinetic energy. This in turn causes a higher entropy increase and consequently a lower stagnation pressure increase for the same total enthalpy increase. At the same Mach number, the stagnation pressure is directly related to static pressure by Eq. (2.6.5), or $Pt = p\cdot f(M)$. So we can also conclude that the greater the load factor is, the more static pressure is lost in the isothermal case.
In the next section, we will analyze the option of fixed cross sectional area instead of fixed temperature for several different load factors.

3.2 Faraday Accelerators with Fixed Area and Load Factors

The discussion of this section will be similar to the previous section, because the plasma flows behave in a similar way except that the temperature takes the place of area. For $K = 1.1$ and $K = 1.15$ (Fig. 3.2.1-3.2.2), all the property variations are gradual and nearly linear (note that the unit of power in Fig. 3.2.1 is 10kW). At $K = 1.2$, the solution is terminated before $x = 0.7$ m, whereas $K = 1.3$, the computation diverges even earlier at $x = 0.3$ m (Figs. 3.2.3-3.2.4). Compared to the previous section, the obvious difference of these cases is that, for the same value $K$ and the position $x$, less power can be added to the fluid, consequently lower Mach numbers are achieved, and the pressure decrease is lower than the corresponding case with fixed temperature. Hence for the fixed area option, it is harder to accelerate the fluid.

The comparisons of these cases are shown in Figs. 3.2.6 and 3.2.7. The temperature, pressure and power input (or Mach number) variations with respect to Mach number (or power) are shown. As the load factor is increased, at the same Mach number (Fig. 3.2.6), a larger input power, correspondingly higher temperature and pressure, is obtained. Certainly, the higher load factor cases also
experience more Joule dissipation (section 2.8) although the static pressure is higher.

Let us examine the charts carefully with respect to the distributions of temperature. Although the Mach number increases in all cases, the temperature decreases for small load factor, decreases then increases for medium load factors, and continually increases for high load factor. A similar situation occurs for the area distributions with the fixed temperature cases also (Fig. 3.1.6). From these results, an exciting and important idea is concluded that both temperature and cross sectional area would be able to be specified under a certain load factor distribution. For example, the constant temperature and constant area channel flow with a certain load factor distribution should be possible. This case will be analyzed in the next section.

The relations between transport properties ($\sigma$ and $\beta$) and pressure are shown in Fig. 3.1.5. However, the electric conductivity $\sigma$ is a very sensitive function of temperature, which is shown in Fig. 3.2.5 with data adopted from the case shown in Fig. 3.2.3. For instance, at $p = 30$ kPa, $\sigma$ is about $132$ A/(V m) if $T = 2500$ K (Fig. 3.1.5), but $\sigma$ approaches $440$ A/(V m) when the temperature is increased to $2800$ K.
3.3 Faraday Accelerators with Constant Temperature and Area

From the manufacturing viewpoint, the best MHD channel contour is the constant area duct. The second one would be a channel with a slight linear deviation, which will be discussed on next section. The computer code of Ref. 4, modified by the author with the secant iteration method, makes it possible to specify both temperature and area variations. For this case, the electric field $E_Y$, which depends on speed and load factor as $E_Y = K u B_z$, is calculated step by step by iteration under the constraints of specified area and temperature.

Figs. 3.3.1 to 3.3.3 show the flow property distributions for the constant area ($A = .00075$ m$^2$), isothermal ($T = 2500$ K) accelerators with $B_z = 2$ T, 3 T and 4 T respectively. The improvement in performance with this design option was exceptional. The magnetic field $B_z$ is the only controllable variable now. The Mach number is increased from about 3.3 with $B_z = 2$ T to 6 with $B_z = 4$ T. For the purpose of easily reading the load factor $K$, it was shown in the figures by $10^{(K-1)}$. The load factors decrease rapidly toward a value close to unity. An even more exciting chart shown in Fig. 3.3.5 is the comparison of the flow properties. All the distributions of the same properties seem to be coincident. The only exception is the load factor at the inlet, which is because of the inaccurately estimated initial value for the iteration of the numerical computation. If the small deviation of the power curves,
which shows that the case with larger magnetic field strength consumed a little less power, can be neglected, we could conclude that the flow property distributions (with respect to Mach number) are independent of the magnetic field for the constant-area isothermal cases.

We have to keep in mind that the power supply capacity is not unlimited. Practically, an 800 kW input power level is about the limit of the power supply capacity at the Aerodynamics Research Center (ARC) of UTA, but in this design analysis, we would still like to investigate cases with larger power supply since it may be possible in the future to expand the capacity. The case shown in Fig. 3.3.2 is one of the best designs obtained with the current design constraints.

An interesting phenomenon shown in Fig. 3.3.4 is the computational result for the case of $B_z = 4.42$ T. Every curve becomes a horizontal line after the fourth integration step ($x = 0.04$ m). The reason is that the computational iteration picks up another solution which also satisfies the constant-area, constant-temperature restraint. That is the case of $K = 1$ — a flow without anything changed. For the cases with $B_z$ greater than 4.42 T, the results are similar but become trivial earlier.
3.4 Faraday Accelerators with Constant Temperature and Linearly Increasing Area

The flow properties are very sensitive to increasing area. Fig. 3.4.1 shows the results for a case with a linearly increasing area. The rate of area increase is very small such that the exit area is only 1.44 times of the inlet, and the half-angle of the walls is only about 0.157°. Both the Mach number and power are significantly increased compared to the constant area case. Moreover, for the case with exit-inlet area ratio $\frac{A_e}{A_i} = 2$, as shown in Fig. 3.4.2, the maximum Mach number is increased to about 8 with an input power as high as 2413kW. Fig. 3.4.3, which shows a comparison of performance for different area ratio, gives us a clear indication that the cases with greater area ratio $\frac{A_e}{A_i}$ are accompanied with higher load factor and more "static pressure drop."

The maximum magnetic field for which the numerical computation can be achieved for the entire channel (1 m in length) is only 0.2 T when the area is linearly increased to eight times that of the inlet area ($A_e/A_i = 8$, Fig. 3.4.4). The load factor K is greater than eight at the channel entrance, which causes a tremendous Joule dissipation and static pressure drop. As shown in Fig. 3.4.5, with higher input power because of the lower magnetic field ($B = 0.2$), the static pressure curve diverges significantly.

From the results discussed in this section, an idea is formulated for further research that suggests there should be an optimum area as well as magnetic field distribution such that the
Joule dissipation and the input power can be minimized to reach the same Mach number at the exit.

3.5 Hall Accelerators

From the operational point of view, the Hall accelerator is more desirable than the Faraday accelerator because only one power supply is needed rather than a series of isolated power supplies. Nevertheless, the computational results are quite frustrating in that no case could be found that match the design criteria.

Fig. 3.5.1 is a typical result for the constant temperature Hall "accelerator" with fixed axial current \( I_x = 80 \, \text{A} \), which always transitions to a Hall generator mode of operation after a certain distance from the inlet. As \( x \) is increased, both \( J_y \) and \( E_x \) decrease linearly and become negative at about \( x = 0.28 \, \text{m} \) for \( J_y \) and \( x = 0.35 \, \text{m} \) for \( E_x \). Thus the accelerator transitions to a decelerator mode when \( J_y \) becomes negative, because the negative \( J_y \) induces a negative body force (see section 2.9). Quickly, after a short displacement, it transitions to the generator mode when \( E_x \) also becomes negative, which means that electrical energy is now being extracted (\( P_d = J_x E_x < 0 \)).

The reason for this behavior can be found from an examination of the local performance chart for a Hall converter shown in Fig. 2.4. First of all, let us review the electromagnetic equations for a Hall converter, which were developed in section 2.5. They are

\[
(2.5.8) \quad J_y = \sigma \left( -u B_z + \beta E_x \right) / (1 + \beta^2)
\]
(2.5.9) \[ Ex = Jx \left[ \frac{(1 + \beta^2)}{\sigma} \right] - \beta u Bz \]

For a Hall accelerator with a specific value of axial current \( I_x \) specified, the current density \( J_x \) is also constant if we assume that the area is constant. From Eqs. (2.5.8) and (2.5.9), when the speed \( u \) is increased, both \( J_y \) and \( Ex \) are decreased if the influence of the transport properties (\( \sigma \) and \( \beta \)) are also neglected. In Fig.2.4, the intersections of the \( Ex \)-line and \( Jx \)-line, and the \( Jy \)-line and \( Jx \)-line are \( \sigma u Bz/\beta^{-1} + \beta \) and \( \sigma u Bz/\beta \) respectively (set \( Ex = 0 \) in Eq. (2.5.9), and \( Jy = 0 \) in Eq. (2.5.8)). When the speed \( u \) increases, both of the \( Ex \)- and \( Jy \)-line would move toward the right because the values of both intersection points (with respect to \( Jx \)-line) are increased. Consequently, for a fixed point on the \( Jx \)-line (fixed \( I_x \)) which is in the accelerator region, when \( u \) increases, it will first be overtaken by the decelerator region, and then by the generator region. The power-line shown in Fig. 3.5.1 also demonstrates this phenomenon clearly, since it varies continuously with a maximum value at the same axial position as \( Ex = 0 \), where the converter transitions to the generator mode.

All the other Hall accelerator cases are similar to this one. For the cases with the fixed area option (Figs. 3.5.3-3.5.5), the situation is even worse. In Figs. 3.5.3 and 3.5.4, the Mach number decreases throughout the channel. All the cases diverge more rapidly when either the B-field or specified axial current \( I_x \) is increased.
In conclusion, the Hall accelerator does not appear to be acceptable for the current design requirements. The feasibility is negative.

3.6 Wind Tunnel Performance

After the discussion of the various MHD accelerator design options in sections 3.1 to 3.5, we conclude that, for the specified design criteria, the best design option is the constant-area isothermal Faraday accelerator. In this section, the resultant wind tunnel performance (Figs. 3.6.1 and 3.6.2) is shown for two of these designs which represent cases with upper limits of the anticipated power supply capability. They are the cases discussed in section 3.3 with $B = 2 \, \text{T}$, $P = 338 \, \text{kW}$ and $B = 4 \, \text{T}$, $P = 1396 \, \text{kW}$ (Figs. 3.3.1 and 3.3.3). The current power supply capability of ARC can already cover the former case. Certainly, there is no definite limitation for either the maximum power supply, which depends on the designed power supply capacity for each wind tunnel, or the minimum power supply, which is large enough so that the MHD augmentation is necessary.

Figs. 3.6.1 and 3.6.2 show the charts of the unit Reynolds number, $\rho u/\mu$, vs. Mach number for logarithmic scale and linear scale. The value of $\mu$, the coefficient of viscosity, is taken from Ref. 8. The $\rho u/\mu$ lines are horizontal for the flow within the MHD accelerator, because $\rho u = \text{constant}$ for a constant-area steady flow and temperature is also assumed constant. Then, through the post nozzle, the flow is accelerated as a "frozen" isentropic expansion flow. The
word frozen is emphasized here because the thermally frozen expansion is assumed for the high speed flow even though there is no chemical dissociation, i.e., the assumption of the CPG is valid. The importance of the diffuser and vacuum system is obvious on the charts. The maximum Mach number depends on the minimum static pressure which could be achieved in the test section.

For example, if the test section static pressure can be maintained as low as \( P = 10^2 \text{ Pa} (10^{-3} \text{ atm}) \), the Mach number would vary from about \( M = 8 \) for the case with the accelerator with \( B = 2 \text{T} \) to \( M = 12 \) for the case with \( B = 4 \text{T} \). This is a significant extension in the capability of the conventional arc-heated wind tunnel, particularly when it is considered that the test section flow should be in equilibrium with MHD augmentation. In general, the test section flow would not be in equilibrium if sufficient energy is added in the arc heater to enable expansion to these Mach numbers.
CHAPTER 4

CONCLUSIONS AND RECOMMENDATIONS

The performance of an MHD accelerator for augmentation of an electric arc heated hypersonic wind tunnel was analyzed. Although the Hall accelerator was not feasible for the current design criteria, the Faraday accelerator with both temperature and area fixed provides an adequate design. Use of MHD augmentation was shown to greatly expand the equilibrium performance envelope of the facility.

Even though a specific distribution of area may produce better performance, as mentioned earlier, the constant area channel is still the simplest one for practical construction of the MHD converter.

The real MHD channel flow is much more complicated because of the boundary layer effect encountered. Not only must the wall contours be adjusted to compensate for boundary layer blockage, the boundary layer voltage drop must be accounted for in sizing the power supply. These corrections are generally straightforward, and the code developed in Ref. 4 can be used to modify the current design for boundary layer effect.
FIGURES
Fig. 1.1 General feature of a MHD augmented arc-heated hypersonic wind tunnel (from Ref. 1)
Fig. 2.1  Linear MHD channel flow geometry and coordinates  
(from  Ref. 4)
a. Faraday Converter

b. Hall converter

Fig. 2.2  Linear Faraday and Hall MHD converters
Fig. 2.3 Local electromagnetic performance of a Faraday converter
Fig. 2.4  Local electromagnetic performance of a Hall converter
Fig. 3.1.1 Faraday accelerator flow property distributions with $T = 2500K$, $B = 2T$, $K = 1.1$
Fig. 3.1.2 Faraday accelerator flow property distributions with $T = 2500K$, $B = 2T$, $K = 1.15$
Fig. 3.1.3  Faraday accelerator flow property distributions with $T = 2500\, K$, $B = 2T$, $K = 1.2$
Fig. 3.1.4  Faraday accelerator flow property distributions with $T = 2500K$, $B = 2T$, $K = 1.3$
Fig. 3.1.5 Relations between transport properties, $\sigma$ and $\beta$, and pressure at $T = 2500K$
Fig. 3.1.6 Comparison of flow properties from $K = 1.1$ to $K = 1.3$ with $T = 2500K$, $B = 2T$
Fig. 3.2.1 Faraday accelerator flow property distributions with $A = 0.00075m^2$, $B = 2T$, $K = 1.1$
Fig. 3.2.2 Faraday accelerator flow property distributions with $A = 0.00075 \text{m}^2$, $B = 2T$, $K = 1.15$
Fig. 3.2.3 Faraday accelerator flow property distributions with $A = 0.00075m^2$, $B = 2T$, $K = 1.2$
Fig. 3.2.4 Faraday accelerator flow property distributions
with $A = 0.00075 m^2$, $B = 2T$, $K = 1.3$
Fig. 3.2.5 Relations of transport properties, pressure, and temperature
Fig. 3.2.6  Comparison of the flow properties (vs. M) from $K = 1.1$ to $K = 1.3$ with $A = 0.00075\text{m}^2$, $B = 2T$
Fig. 3.3.1 Faraday accelerator flow property distributions with $A = 0.00075\text{m}^2$, $T = 2500\text{K}$, $B = 2\text{T}$.
Fig. 3.3.2 Faraday accelerator flow property distributions
with $A = 0.00075\text{m}^2$, $T = 2500\text{K}$, $B = 3\text{T}$
Fig. 3.3.3 Faraday accelerator flow property distributions with $A = 0.00075m^2$, $T = 2500K$, $B = 4T$
Fig. 3.3.4 Faraday accelerator flow property distributions with $A = 0.00075 \text{m}^2$, $T = 2500 \text{K}$, $B = 4.42 \text{T}$
Fig. 3.3.5 Comparison of flow properties from $B = 2T$ to $B = 4T$ with $A = 0.00075m^2$, $T = 2500K$
Fig. 3.4.1 Faraday accelerator flow property distributions with \( \frac{Ae}{Ai} = 1.44 \), \( T = 2500K \), \( B = 3T \)
Fig. 3.4.2 Faraday accelerator flow property distributions with $Ae/Ai = 2$, $T = 2500K$, $B = 3T$
Fig. 3.4.3 Comparison of flow properties between $Ae/Ai = 1, 1.44$ and 2, with $T = 2500K$, $B = 3T$
Fig. 3.4.4 Faraday accelerator flow property distributions with $\frac{Ae}{Ai} = 8$, $T = 2500K$, $B = 0.2T$
Fig. 3.4.5 Comparison of flow properties between $\frac{A_e}{A_i} = 1$ and $\frac{A_e}{A_i} = 8$ with $T = 2500K$, $B = 2T$ and $0.2T$ respectively.
Fig. 3.5.1 Hall accelerator flow property distributions
with $T = 2500K$, $B = 2T$, $I_x = 80A$
Fig. 3.5.2 Hall accelerator flow property distributions with $T = 2500K$, $B = 2T$, $Ix = 150A$
Fig. 3.5.3 Hall accelerator flow property distributions
with $A = 0.00075 m^2$, $B = 1 T$, $I_x = 150 A$
Fig. 3.5.4 Hall accelerator flow property distributions
with $A = 0.00075 \text{m}^2$, $B = 1 \text{T}$, $I_x = 400 \text{A}$
Fig. 3.5.5  Hall accelerator flow property distributions
with  $A = 0.00075\text{m}^2$, $B = 2\text{T}$, $I_x = 150\text{A}$
Fig. 3.6.1 MHD augmented hypersonic wind tunnel performance (logarithm scale)
Fig. 3.6.2 MHD augmented hypersonic wind tunnel performance (linear scale)
REFERENCES


